Future of Lattice Calculations for $b$ Physics

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Outline

Introduction

Required parameters

Target simulations

$b$ physics

Conclusions
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Will we be able to calculate hadronic parameters for $b$-physics with 1% or a few % precision by 2015?

Consider

- Required simulation parameters
- Scaling formulae and computational costs
- Requirements for $b$-physics

I rely heavily on:

- S Sharpe, *Weak Decays of Light Hadrons*, LQCD Present and Future, Orsay 2004
- V Lubicz, *CKM Fit and Lattice QCD*, SuperB IV, Monte Porzio Catone 2006
Errors in lattice calculations

- Statistical
- Systematic
Errors in lattice calculations

- **Statistical**
  - Arise from Monte Carlo evaluation of functional integrals
  - Rule of thumb: about 100 *independent* configurations for
    \[ \sim 1\% \] statistical error
  - . . . but depends on quantity studied, lattice volume, exact formulation of LQCD used

- **Systematic**
Errors in lattice calculations

- **Statistical**
  - Arise from Monte Carlo evaluation of functional integrals
  - Rule of thumb: about 100 independent configurations for \( \sim 1\% \) statistical error
  - … but depends on quantity studied, lattice volume, exact formulation of LQCD used

- **Systematic**
  - Discretisation and continuum extrapolation (\( \alpha \neq 0 \))
  - Light quarks: chiral extrapolation (\( m_l \rightarrow m_{ud} \))
  - Finite volume (\( L \neq \infty \))
  - Heavy quarks (\( m_Q \rightarrow m_{c,b} \))
  - Renormalisation constants (matching lattice to continuum)
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Lattice spacing

Estimate from Sharpe, LQCD present and future, Orsay 2004

Assume using $O(\alpha)$-improved action for observable $\mathcal{O}$

$$\mathcal{O}_{\text{latt}} = \mathcal{O}_{\text{phys}} \left[ 1 + c_2(a\Lambda)^2 + c_n(a\Lambda)^n + \cdots \right]$$

- assume $c_2, c_n$ are $O(1)$
- $n = 3, 4$ depending on action used
- $\Lambda \sim \Lambda_{\text{QCD}}$ for light quarks
- $\Lambda \sim m_Q$ for heavy quarks $Q$ (so more work needed to avoid lattice artefacts ... see below)

Simulate at $a_{\text{min}}$ and $\sqrt{2}a_{\text{min}}$ and extrapolate linearly in $a^2$. Resulting error:

$$\frac{\Delta \mathcal{O}_{\text{phys}}}{\mathcal{O}_{\text{phys}}} \approx c_n(2^{n/2} - 2)(a_{\text{min}}\Lambda)^n$$
Lattice spacing estimates

\[
\frac{\Delta O_{\text{phys}}}{O_{\text{phys}}} \approx c_n (2^{n/2} - 2) (a_{\text{min}} \Lambda)^n
\]

For 1\% error (taking \(c_n = 1\))

<table>
<thead>
<tr>
<th>(\Lambda)</th>
<th>0.5 GeV</th>
<th>0.8 GeV</th>
<th>1.5 GeV</th>
<th>4.5 GeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a_{\text{min}}(n = 3))</td>
<td>0.091 fm</td>
<td>0.057 fm</td>
<td>0.030 fm</td>
<td>0.010 fm</td>
</tr>
<tr>
<td>(a_{\text{min}}(n = 4))</td>
<td>0.105 fm</td>
<td>0.066 fm</td>
<td>0.035 fm</td>
<td>0.012 fm</td>
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- Current lattice spacings 0.05 fm \(\leq a \leq 0.13\) fm
- OK for light quarks
- Daunting for charm
- Need effective theories for \(b\)
Minimum light quark mass

Estimate from ChPT:

\[ \mathcal{O}_{\text{latt}} = \mathcal{O}_{\text{phys}} \left[ 1 + c_2 \left( \frac{m_\pi}{m_\rho} \right)^2 + c_4 \left( \frac{m_\pi}{m_\rho} \right)^4 + \cdots \right] \]

- Assume \( c_n \) are \( O(1) \)
- Simulate at \( R_{\text{min}} = (m_\pi/m_\rho)_{\text{min}} \) and \( \sqrt{2} R_{\text{min}} \) and extrapolute linearly in \( R^2 \)
- Resulting error:

\[ \frac{\Delta \mathcal{O}_{\text{phys}}}{\mathcal{O}_{\text{phys}}} \approx 2c_2 \left( \frac{m_\pi}{m_\rho} \right)^4 \]

- For 1% error (taking \( c_2 = 1 \)):

\[ \left( \frac{m_\pi}{m_\rho} \right)_{\text{min}} \approx 0.27 \quad \text{or} \quad \frac{m_l}{m_s} \approx \frac{1}{11} \quad \text{or} \quad m_\pi,_{\text{min}} \approx 210 \text{ MeV} \]
Finite volume: minimum box size

- FV effects matter when aiming for 1% precision
- Dominant effect from pion loops ⇒ estimate using ChPT
- Example: FV effects in $f_{B_s}/f_{B_d}$ from HMChPT (Arndt, Lin, prd70 014503)
Finite volume effects

- For quantities without final state interactions
  \[ \frac{\Delta \mathcal{O}_{\text{phys}}}{\mathcal{O}_{\text{phys}}} \approx ce^{-m_\pi L} \]
  where \( c \) is \( O(1) \), but depends on quantity calculated
- For 1% error (with \( c = 1 \))
  \[ m_\pi L \approx 4.6 \]
- If \( m_\pi = 200 \text{ MeV} \) then
  \[ L \approx 4.5 \text{ fm} \]
Heavy quarks

- From discussion above, a relativistic $b$ quark would require $am_b \ll 1$, say
  
  $$a \approx 0.01 \text{ fm}$$

- This is *too small* even for Pflop computers
- Various approaches:
  - effective theories
  - interpolation between static limit and charm region
- ... see later
Renormalisation (Matching)

\[ \mathcal{O}^R(\mu) = Z(\alpha \mu, g)\mathcal{O}^{\text{latt}}(\alpha) \]

- Nonperturbative (points) versus perturbative (curves) renormalisation of static-light axial current
- \( N_f = 2 \)
- PT off by \( \sim 5\% \) at hadronic scale
- Use NPR for 1\% precision
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Target simulations: aiming at 1% precision

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<td>$N_{\text{conf}}$</td>
<td>120</td>
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<td>$\alpha$</td>
<td>1/20 fm</td>
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<td>$a^{-1}$</td>
<td>$\approx 4$ GeV</td>
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<tr>
<td>$m_l/m_s$</td>
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<td>$m_\pi$</td>
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<td>$\text{Vol}$</td>
<td>$90^3 \times 180$</td>
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- Tough for charm; $b$ not directly simulated on full-size lattice
- Are such simulations feasible? Compare computer power to estimated computational cost
Computer power

LQCD
- 1–10 Tflop/s today
- 1–10 Pflop/s 2015
Varieties of fermions

<table>
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JMF BNM2008 18/35
### Varieties of fermions

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Varieties of fermions

Wilson
- standard
- $O(a)$-improved
- twisted mass

Staggered
- first to reach light masses: $m_l/m_s \sim 1/8$
- “ugly” (Sharpe, hep-lat/0610094)
- Staggered $\Rightarrow$ 4 tastes per flavour
- Reduced to one by 4th root of quark determinant
- Rooted staggered fermions unphysical for $a \neq 0$, but go over to single-taste theory in limit $a \to 0$.
- rS$\chi$PT: complicated fits with unphysical effects included in fit

Ginsparg-Wilson

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<td>• good chiral properties</td>
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<td></td>
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<td>• 10–30 price hike</td>
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Algorithmic progress

Tremendous progress in C21.

- **RHMC** (Clark–Kennedy, NPBPS129 850, PRL98 051601)
- **Mass preconditioning** (Hasenbusch, PLB519 177; Urbach et al, CPC174 87)
- **Domain-decomposition** (Del Debbio et al, JHEP02 056)
Algorithmic progress

Compare 100 configurations of $N_f = 2$, $O(\alpha)$-improved Wilson fermions:

- **2001** (Ukawa, Lattice2001)

  \[ 5 \left( \frac{0.2}{m_l/m_s} \right)^3 \left( \frac{L}{3 \text{ fm}} \right)^5 \left( \frac{0.1 \text{ fm}}{a} \right)^7 \text{ TflopsYr} \]

- **2006** (Del Debbio et al, JHEP02 056): DD-HMC

  \[ 0.05 \left( \frac{0.2}{m_l/m_s} \right) \left( \frac{L}{3 \text{ fm}} \right)^5 \left( \frac{0.1 \text{ fm}}{a} \right)^6 \text{ TflopsYr} \]
100 conf

$L = 2.5\, \text{fm}$

$a = 0.08\, \text{fm}$

$V = 32^3 \times 64$
## Cost estimates: Wilson fermions

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Wilson $0.07$ Pflops yr $0.9$ Pflops yr

- Overhead for $N_f = 2 + 1$ and generating extra ensembles at larger $a$ and larger $m_l$ is a factor of about 3
- Bigger overhead for GW simulations (with good chiral symmetry)
Cost estimate: DWF fermions

DWF scaling formula (Christ and Jung, Lattice 2007)

\[
\text{Cost } \propto \left( \frac{L}{\text{fm}} \right)^5 \left( \frac{\text{MeV}}{m_\pi} \right) \left( \frac{\text{fm}}{\alpha} \right)^6 
\times \left\{ C_0 + C_1 \left( \frac{\text{MeV}}{m_K} \right)^2 \left( \frac{\text{fm}}{\alpha} \right) + C_2 \left( \frac{\text{MeV}}{m_\pi} \right)^2 \left( \frac{\alpha}{\text{fm}} \right)^2 \right\}
\]

- About 1.5 Pflops yr for the light quark target simulation
- May not need such small \( \alpha \) (0.05 fm) for DWF
- Physics projects may demand larger volumes? \( (L > 4.5 \text{ fm}) \)
- RBC–UKQCD able to do this around 2011?
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Simulating a relativistic $b$-quark with 1% errors needs $a \sim 0.01 \text{ fm}$

- Cost scales as $a^{-6}$ or $a^{-7}$
- Prohibitive even for Pflops computers if you want a big ($L \approx 4.5 \text{ fm}$) lattice as well

Charm physics is feasible with Wilson fermions

- For $a = 0.033 \text{ fm}$, cost for 120 configurations $\sim 0.9 \text{ Pflops yr}$
Lattice $b$-physics: complementary approaches

- Simulate relativistic quarks in charm region and extrapolate to $b$
- Effective theories
  - HQET: substantial progress in nonperturbative renormalisation, use of static-link fattening and inclusion of $O(\Lambda_{QCD}/m_b)$ corrections
  - NRQCD or Fermilab/Tsukuba (RHQ) actions
- Finite-volume and step-scaling approach of Rome-II group

\[ \mathcal{O}(L_\infty) = \mathcal{O}(L_0) \frac{\mathcal{O}(L_1)}{\mathcal{O}(L_0)} \cdots \frac{\mathcal{O}(L_N)}{\mathcal{O}(L_{N-1})} \]

$L_0$ small enough to allow $a \approx 0.01 \text{ fm}$ and $L_N \sim L_\infty$ (last factor is 1 to required precision)
- Step-scaling also used for nonperturbative renormalisation of HQET (ALPHA) and RHQ (Christ & Lin, prd76 074505/6)
Current status: interpolation

- ALPHA have implemented interpolation between static results and relativistic charm-scale results.
- ALPHA and Rome-II have combined static and step-scaling results.
- Both reach 3% precision for $f_{B_s}$.
- ... but still in quenched approximation and consider $f_{B_s}$ so no chiral extrapolation.
Interpolation: static and relativistic

$f_{B_s} = 193(6) \text{ MeV}$

- $\alpha \to 0$ before $1/m_{PS}$ interpolation
- still quenched
- no chiral extrapolation

Della Morte et al, arXiv:0710.2201
Interpolation: static and relativistic/step-scaling

$f_{B_s} = 193(6)\,\text{MeV}$

$f_{B_s} = 191(6)\,\text{MeV}$

- $\alpha \to 0$ before $1/m_{PS}$ interpolation
- still quenched
- no chiral extrapolation

Della Morte et al, arXiv:0710.2201
Guazzini et al, arXiv:0710.2229
Heavy-to-heavy semileptonic decay: $B \rightarrow Dl\nu$

- Rome-II step-scaling method plus twisted BCs
- Lattice data normalized to experiment at $\omega = 1.2$
- 2% error on $G(\omega=1) \ldots$ quenched
Heavy-to-heavy semileptonic decay: $B \rightarrow D^* l \nu$

Extract $h_{A_1}$ directly from double ratio:

$$|h_{A_1}(1)|^2 = \frac{\langle D^* | \bar{c} \gamma_j \gamma_5 b | \bar{B} \rangle \langle \bar{B} | \bar{b} \gamma_j \gamma_5 c | D^* \rangle}{\langle D^* | \bar{c} \gamma_4 c | D^* \rangle \langle \bar{B} | \bar{b} \gamma_4 b | \bar{B} \rangle}$$

Plot: $h_{A_1}(1)$ vs $m_\pi^2$

- 2 + 1 improved staggered ⇒ rSχPT fit
- Fermilab heavy quarks
- Quote 2.3% error

Laiho, arXiv:0710.1111
Current status: $B \rightarrow \pi$ semileptonic decays

- Results from Fermilab and HPQCD using different effective theories
  - Fermilab: Fermilab action (not final . . .)
  - HPQCD: NRQCD ($\text{prd73 074502, prd75 119906(E)}$)
- Results have come into agreement
- . . . but, based on same gauge field ensembles
- HPQCD: biggest errors from chiral extrapolation and perturbative matching
- $\sim 14\%$ error on form factors in $q^2 \geq 16 \text{GeV}^2$
- Need confirmation from other approaches
$f_{+,0}(q^2)$

$q^2 / \text{GeV}^2$

$|V_{ub}| = 3.47(29)(03) \times 10^{-3}$
$f_+(0) = 0.245(23)$
$|V_{ub}|f_+(0) = 8.5(8) \times 10^{-4}$

JMF & Nieves, prd76 031302
**b-physics prognosis**

- Best results likely from combining extrapolation from $m_Q \approx m_c$ with effective theory results (including $\Lambda_{\text{QCD}}/m_b$ corrections)
- Few % precision requires nonperturbative renormalisation: this is being done for HQET
- Medium term: look for agreement between different approaches (HQET, NRQCD, Fermilab/Tsukuba) and study theoretical foundations
- Although quenched approximation has been banished from light quark physics, some heavy quark analysis still being developed using quenched ensembles: redo unquenched once methods established
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- Access to Pflops computers with current techniques and knowledge should allow few % precision in $b$-physics.
- Further theoretical and technical advances will likely improve precision further.
- Need all hadrons strongly stable (so not $B \rightarrow \rho$ decays for now).
- For $b$-hadron decays to two-hadron (or higher) states, we need new ideas before we can formulate a numerical approach to evaluating the amplitudes.