Some Thoughts on D^0-$\overline{D}^0$ Mixing

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The oscillation in time of neutral D mesons into their antiparticles, and *vice versa*, commonly called D^0-$\overline{D}^0$ mixing, has been observed by several experiments in a variety of channels during the past two years. The observations of D^0-$\overline{D}^0$ mixing indicate that the physical eigenstates have decay rate differences and/or mass differences greater than expected most naively. In this talk I will discuss the recent experimental results, the extent to which they probe non-perturbative QCD and physics beyond the Standard Model, and issues related to more precise measurements in future experiments.

Presented at the Open SuperBelle Collaboration Meeting, 11 Dec. 2008

Mixing Phenomenology

Neutral D mesons are produced as *flavor eigenstates* D^0 and $\bar{D}^0$ and decay via

$$i\frac{\partial}{\partial t}\begin{pmatrix} D^0(t) \\ \overline{D}^0(t) \end{pmatrix} = \left(\mathbf{M} - \frac{i}{2}\mathbf{\Gamma}\right)\begin{pmatrix} D^0(t) \\ \overline{D}^0(t) \end{pmatrix}$$

as *mass, lifetime eigenstates* D_1, D_2

$$\begin{aligned} |D_1\rangle &= p|D^0\rangle + q|\overline{D}^0\rangle \\ |D_2\rangle &= p|D^0\rangle - q|\overline{D}^0\rangle \end{aligned}$$

where $|q|^2 + |p|^2 = 1$ and

$$\left(\frac{q}{p}\right)^2 = \frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{M_{12} - \frac{i}{2}\Gamma_{12}}$$

D_1, D_2 have masses M_1, M_2 and widths Γ_1, Γ_2

Mixing occurs when there is a *non-zero* mass

$$\Delta M = M_1 - M_2$$

or lifetime difference

$$\Delta\Gamma = \Gamma_1 - \Gamma_2$$

For convenience define, x and y

$$x = \frac{\Delta M}{\Gamma}, \quad y = \frac{\Delta\Gamma}{2\Gamma}$$

where

$$\Gamma = \frac{\Gamma_1 + \Gamma_2}{2}$$

and define the mixing rate

$$R_M = \frac{x^2 + y^2}{2} \quad (< 5 \times 10^{-4})$$

How Mixing is Calculated

$$\left(M - \frac{i}{2}\Gamma\right)_{12} = \frac{1}{2m_D}\langle D^0|\mathcal{H}_w^{\Delta C=2}|\overline{D}^0\rangle + \frac{1}{2m_D}\sum_n \frac{\langle D^0|\mathcal{H}_w^{\Delta C=1}|n\rangle\,\langle n|\mathcal{H}_w^{\Delta C=1}|\overline{D}^0\rangle}{m_D - E_n + i\epsilon}$$

The first term is called the short distance contribution and the second the long distance or dispersive contribution. Assuming the short distance contributions are small, and that CP is conserved, we can express y as an absorptive part of the second term

$$y = \frac{1}{\Gamma_{\rm D}}\sum_n \rho_n \langle \overline{D}^0|\mathcal{H}_w^{\Delta C=1}|n\rangle\langle n|\mathcal{H}_w^{\Delta C=1}|D^0\rangle,$$

where ρ_n is the phase space factor corresponding to the charmless intermediate state $|n\rangle$.

Points of theoretical consensus

- Short distance contributions to x and y are $\ll 10^{-2}$;
- CP is not significantly violated in the Standard Model;
- Large long-distance contributions to x and y may originate in the different phase spaces available for CP-even and CP-odd final states (but not in SM matrix elements); x, $y \sim \mathcal{O}(10^{-2})$ cannot be excluded in the Standard Model;
- New Physics may contribute to mixing at the $y \sim \mathcal{O}(10^{-2})$ level.

Standard Model Mixing Predictions

Box diagram SM charm mixing rate naively expected to be very low (R_M~10^{-10}) (Datta & Kumbhakar)

Z.Phys. C**27**, 515 (1985)

CKM suppression → $|V_{ub}V^*_{cb}|^2$

GIM suppression → $(m^2_s - m^2_d)/m^2_W$

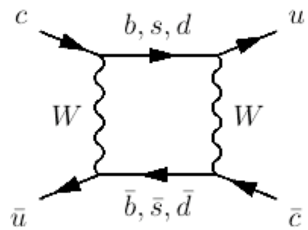
Di-penguin mixing, R_M~10^{-10}

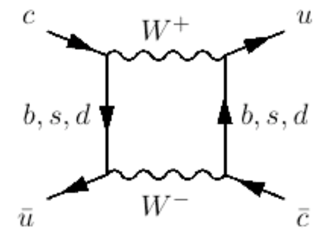
Phys. Rev. D **56**, 1685 (1997)

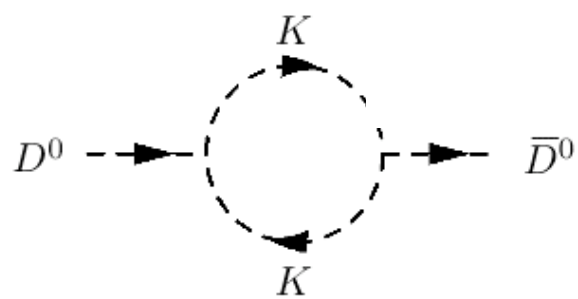
Enhanced rate SM calculations generally due to long-distance contributions:

first discussion, L. Wolfenstein

Phys. Lett. B **164**, 170 (1985)







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Partial History of Long-Distance Calculations

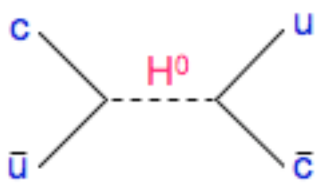
- Early SM calculations indicated long distance contributions produce x<<10^{-2}:
  - x~10^{-3} (dispersive sector)
    - PRD **33**, 179 (1986)
  - x~10^{-5} (HQET)
    - Phys. Lett. B **297**, 353 (1992)
    - Nucl. Phys. **B403**, 605 (1993)
- More recent SM predictions can accommodate x, y ~1% [of opposite sign] (Falk *et al.*)
  - ***x,y*** $\approx \sin^2\theta_C$ x [SU(3) breaking]^2
    - Phys.Rev. D **65**, 054034 (2002)
    - Phys.Rev. D **69**, 114021 (2004)

New Physics Mixing Predictions

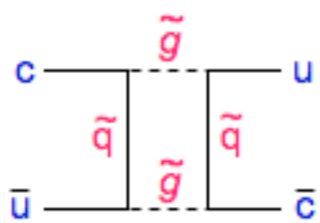
Possible enhancements to mixing due to new particles and interactions in new physics models

Most new physics predictions for x

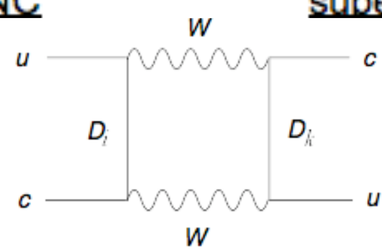
- Extended Higgs, tree-level FCNC
- Fourth generation down-type quarks
- Supersymmetry: gluinos, squarks
- Lepto-quarks



FCNC



supersymmetry



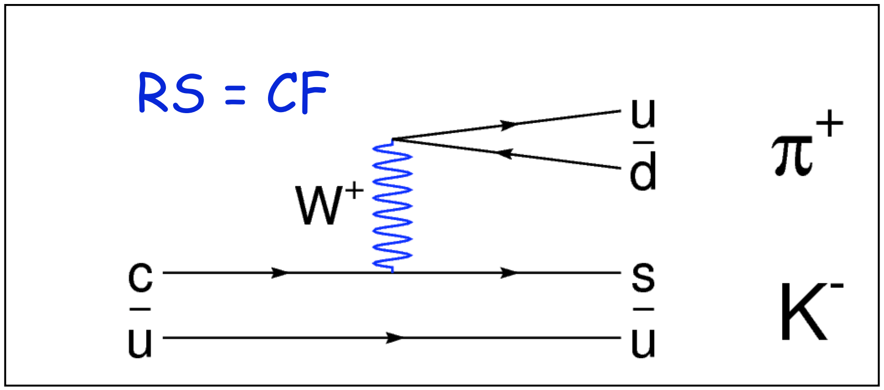
Heavy weak iso-singlet quarks

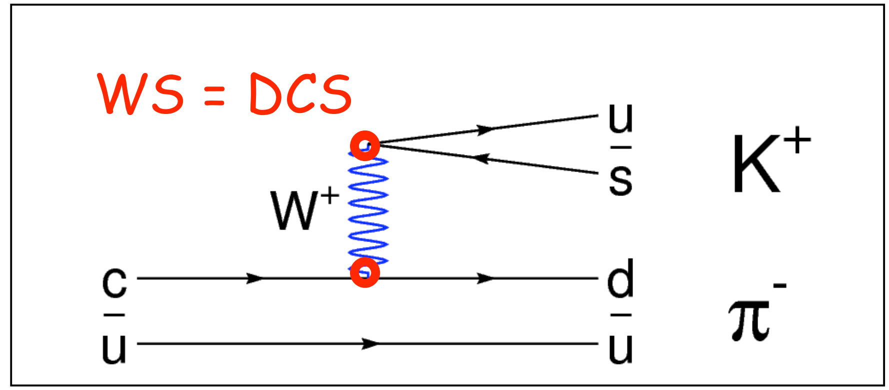
- Large possible SM contributions to mixing require observation of either a CP-violating signal or $|x| >> |y|$ to establish presence of NP
- A recent survey (Phys. Rev. D76, 095009 (2007), arXiv:0705.3650) summarizes models and constraints:

Fourth generation	Vector leptoquarks
Q = -1/3 singlet quark	Flavor-conserving Two-Higgs
Q = +2/3 singlet quark	Flavor-changing neutral Higgs
Little Higgs	Scalar leptoquarks
Generic Z'	MSSM
Left-right symmetric	Supersymmetric alignment

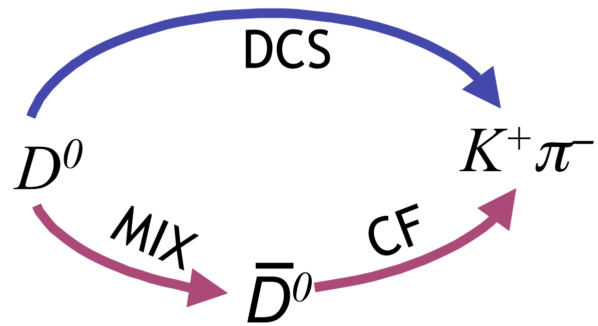
and more

Time-Evolution of $D^0 \to K\pi$ Decays





DCS and mixing amplitudes interfere to give a "quadratic" WS decay rate (x, y « 1):



$$\frac{\Gamma_{WS}(t)}{e^{-t/\tau}} \propto R_D + \sqrt{R_D}\, y' \left(\frac{t}{\tau}\right) + \left(\frac{x'^2 + y'^2}{4}\right)\left(\frac{t}{\tau}\right)^2$$

where $x' = x \cos\delta + y \sin\delta$ $\qquad y' = y\cos\delta - x \sin\delta$

and δ is the phase difference between DCS and CF decays.

Mixing in $D^0 \to K_S \pi^+ \pi^-$

The decay amplitude at time t of an initially produced $|D^0\rangle$ or $|\overline{D}^0\rangle$ can be expressed as

$$\mathcal{M}(m_-^2, m_+^2, t) = \mathcal{A}(m_-^2, m_+^2)\frac{e_1(t)+e_2(t)}{2} + \frac{q}{p}\overline{\mathcal{A}}(m_-^2, m_+^2)\frac{e_1(t)-e_2(t)}{2},$$

$$\overline{\mathcal{M}}(m_-^2, m_+^2, t) = \overline{\mathcal{A}}(m_-^2, m_+^2)\frac{e_1(t)+e_2(t)}{2} + \frac{p}{q}\mathcal{A}(m_-^2, m_+^2)\frac{e_1(t)-e_2(t)}{2}.$$

The time dependence is contained in the terms

$$e_{1,2}(t) = \exp[-i(m_{1,2} - i\Gamma_{1,2}/2)t] .$$

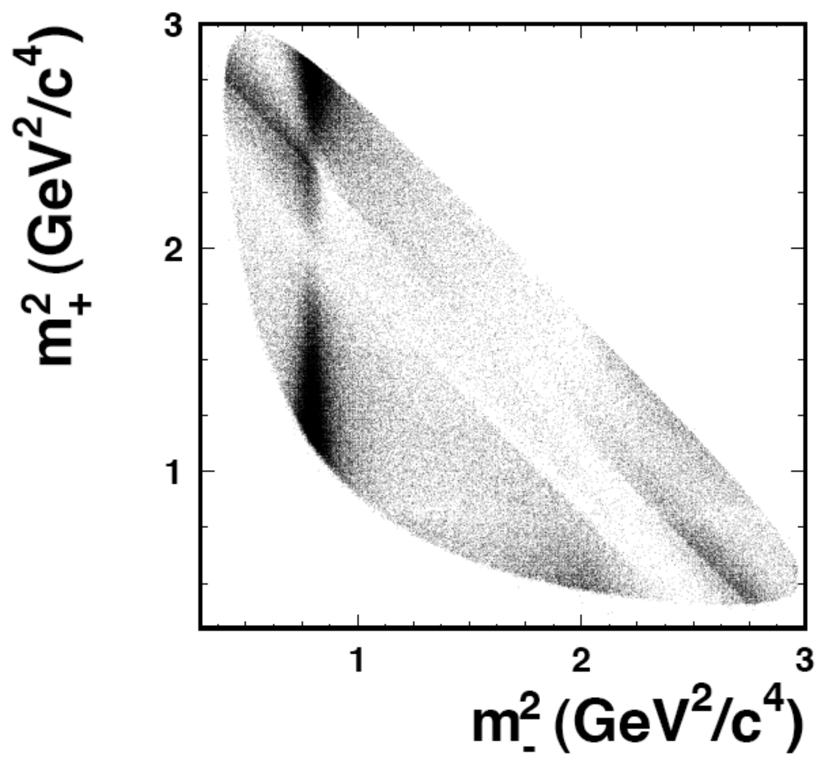
Upon squaring $\mathcal{M}$ and $\overline{\mathcal{M}}$, one obtains decay rates containing terms $\exp(-\Gamma t)\cos(x\Gamma t)$, $\exp(-\Gamma t)\sin(x\Gamma t)$, and $\exp[-(1 \pm y)\Gamma t]$.

Each amplitude is a function of m_+^2 and m_-^2, expressed as a sum of quasi-two-body amplitudes (subscript r) and a constant non-resonant term (subscript NR):

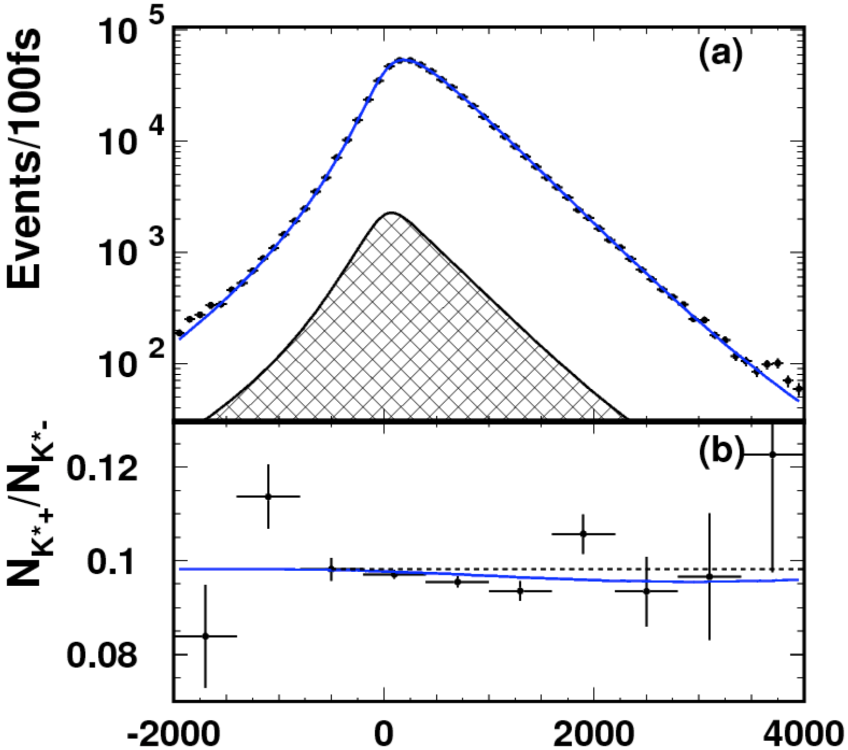
$$\mathcal{A}(m_-^2, m_+^2) = \sum_r a_r e^{i\phi_r}\mathcal{A}_r(m_-^2, m_+^2) + a_{\rm NR}e^{i\phi_{\rm NR}}$$

$$\overline{\mathcal{A}}(m_-^2, m_+^2) = \sum_r \overline{a}_r e^{i\overline{\phi}_r}\mathcal{A}_r(m_+^2, m_-^2) + \overline{a}_{\rm NR}e^{i\overline{\phi}_{\rm NR}}$$

The $\mathcal{A}_r$ are products of Blatt-Weisskopf form factors and relativistic Breit-Wigner functions.



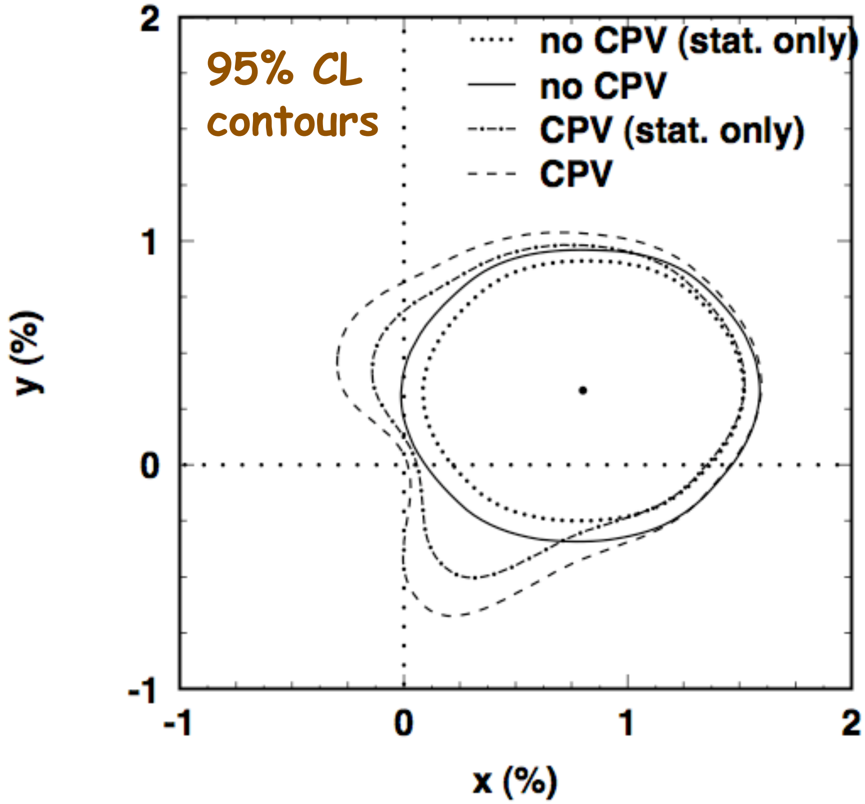
Mixing in $D^0 \rightarrow K_S\pi^+\pi^-$



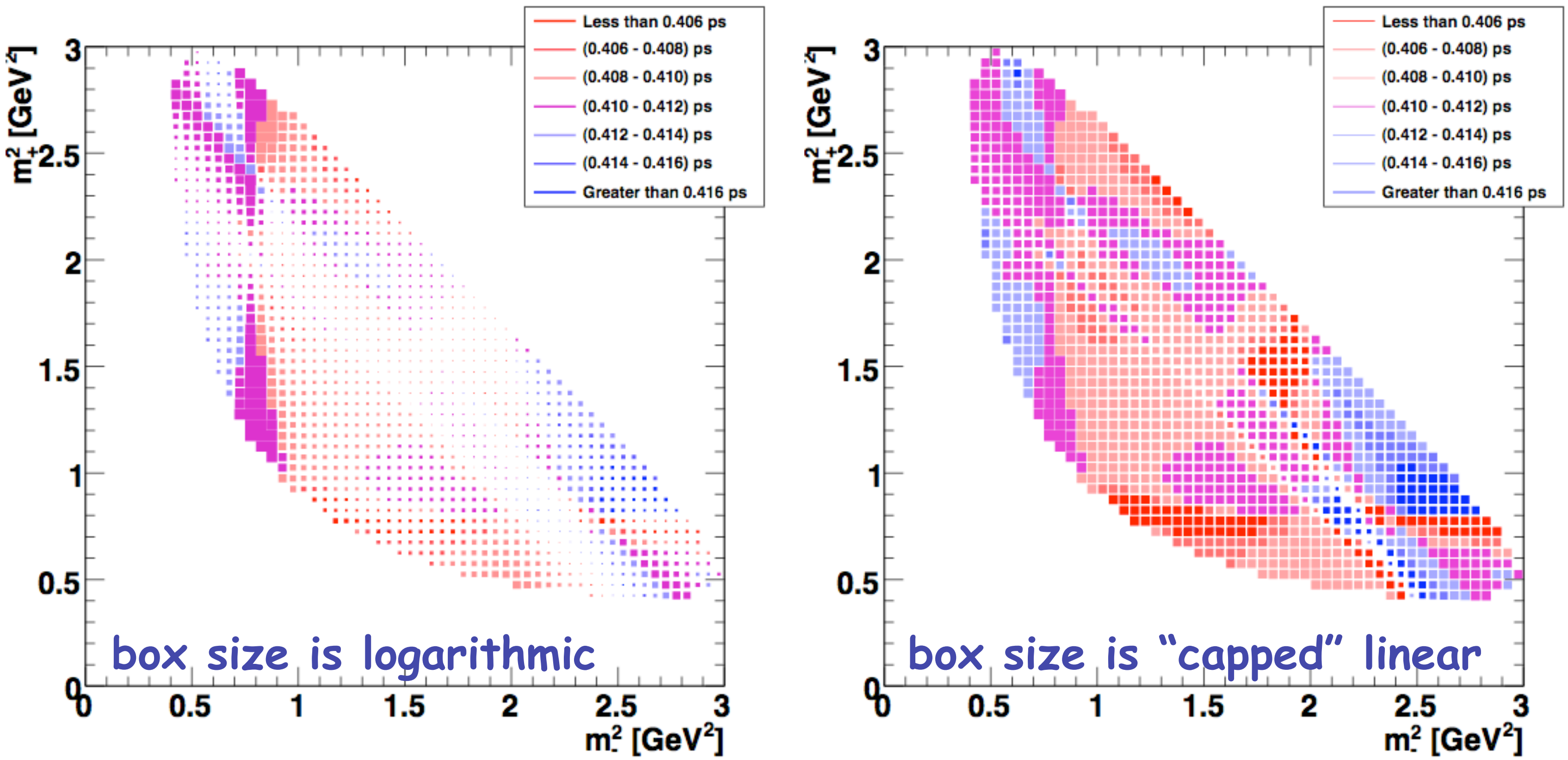
x : $(0.80 \pm 0.35 \pm 0.15)\%$

y : $(0.33 \pm 0.24 \pm 0.14)\%$

(assuming no CP violation)



Time-Dependence in $D^0 \rightarrow K_S\pi^+\pi^-$



These plots illustrate the average decay time as a function of position in the Dalitz plot for (x,y) = (0.8%, 0.3%). The sizes of the boxes reflect the number of entries, and the colors reflect the average decay time.

Mixing in $D^0 \rightarrow K^+\pi^-\pi^0$

"Wrong-sign" decay rate varies across the Dalitz plot:

DCS term

Resonance phase

$$\mathcal{A}(m_{K^-\pi^+}, m_{K^-\pi^0}, t) = e^{-\Gamma t}[|\bar{A}_D|^2 + |\bar{A}_D||A_D|\,(y''\cos\delta_D - x''\sin\delta_l \; |A_D|^2(x''^2 + y''^2)(\Gamma t)^2]$$

Interference term

CF (mixed) term

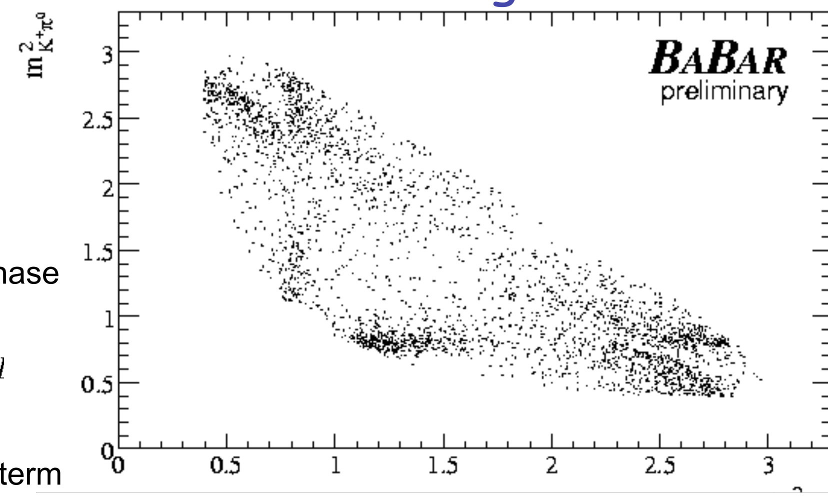
$$\begin{aligned} x'' &= x\cos\delta_{K\pi\pi^0} + y\sin\delta_{K\pi\pi^0} \\ y'' &= y\cos\delta_{K\pi\pi^0} - x\sin\delta_{K\pi\pi^0} \end{aligned}$$

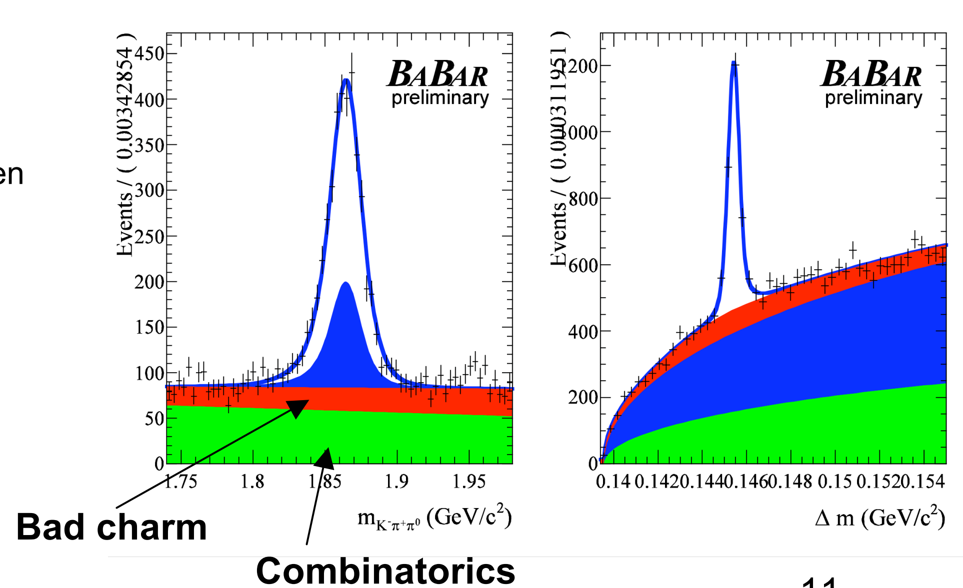
Phase between RS and WS

Subscript D indicates dependence on position in the Dalitz plot.

_ **Yields from 384 fb^{-1}**

1483 ± 56 signal events





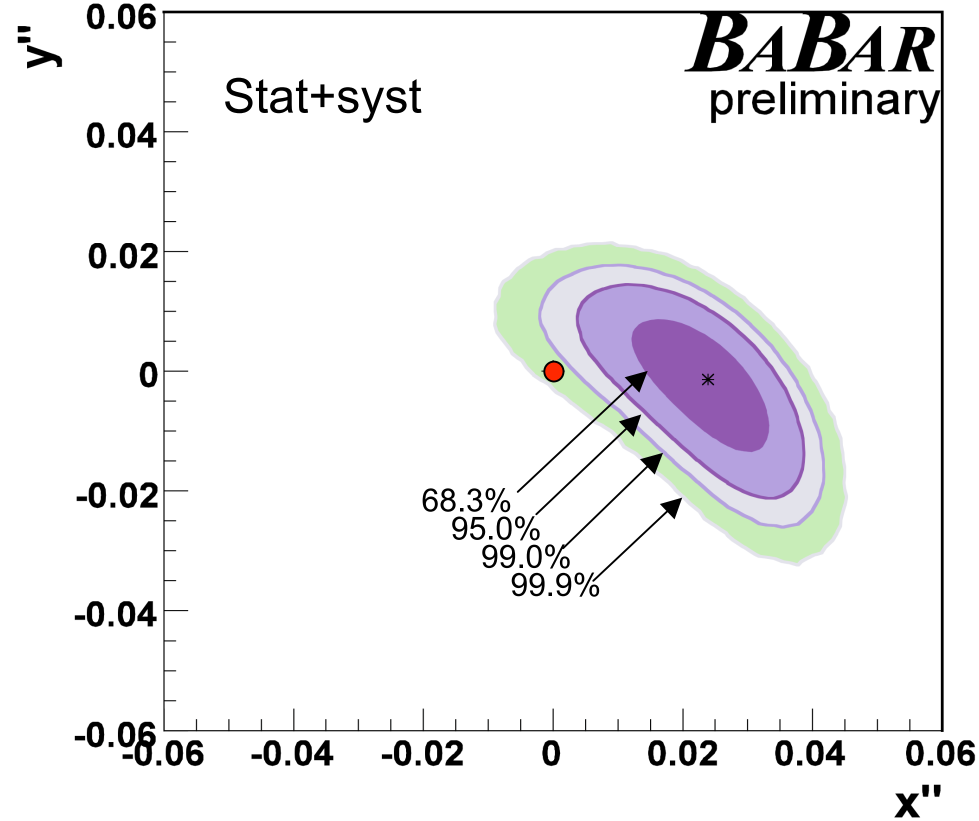
$D^0 \to K^+\pi^-\pi^0$: Results

No mixing is excluded at the 99% confidence level.

x'': (2.39 ± 0.61 ± 0.32) %

y'' : (-0.14 ± 0.60 ± 0.40)%

R_M: (2.9 ± 1.6) × 10^{-4}



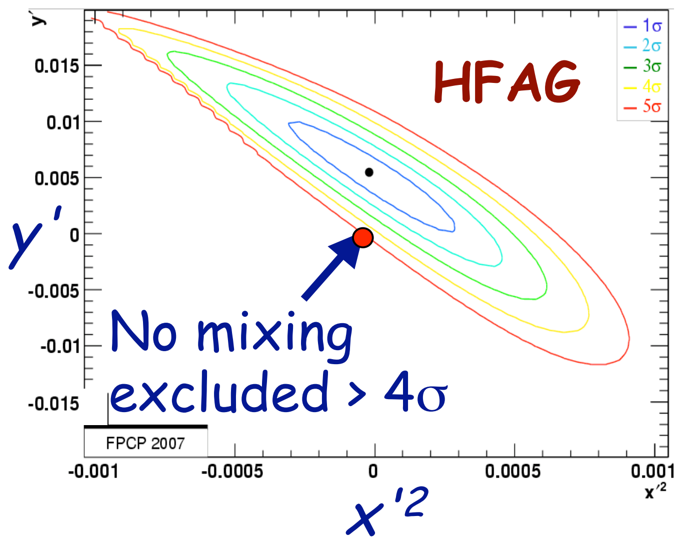
Spring, 2008

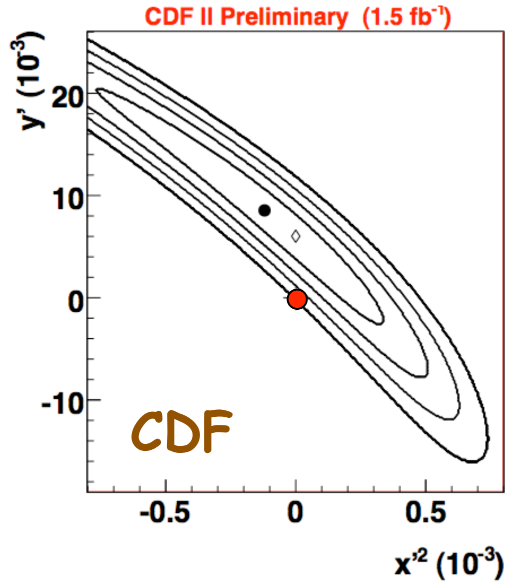
$D^0 \to K\pi$

R_D: $(3.30\ ^{+0.14}_{-0.12}) \times 10^{-3}$

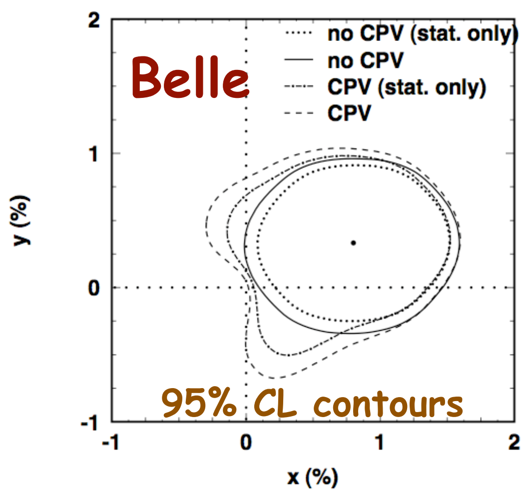
x'^2: $(-0.01 \pm 0.20) \times 10^{-3}$

y' : $(5.5\ ^{+2.8}_{-3.7}) \times 10^{-3}$

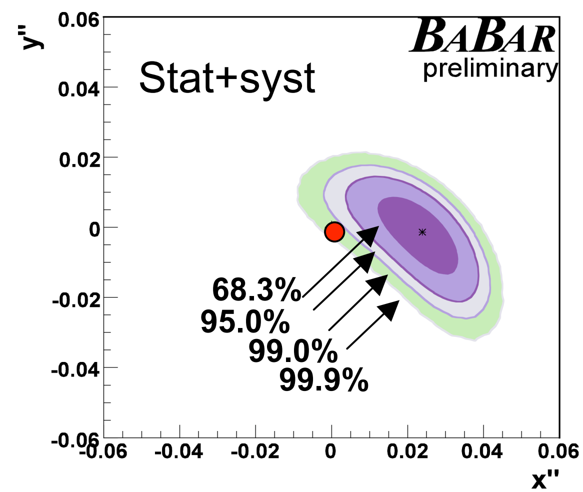




$D^0 \to K_S\pi^+\pi^-$



$D^0 \to K^+\pi^-\pi^0$



$\Delta(\Gamma)$ in $D^0 \to h^+h^-$

BaBar + Belle

$(1.10 \pm 0.27)\%$

$SU(3)$ Breaking and D^0-$\overline{D}^0$ mixing

Falk, Grossman, Ligeti, and Petrov; Phys. Rev D65, 054034 (2002)

$$y = \frac{1}{2\Gamma} \sum_n \rho_n \left[\langle D^0 | \mathcal{H}_w | n \rangle \langle n | \mathcal{H}_w | \overline{D}^0 \rangle + \langle \overline{D}^0 | \mathcal{H}_w | n \rangle \langle n | \mathcal{H}_w | D^0 \rangle \right]$$

$$y = \sum_n \eta_{\text{CKM}}(n)\, \eta_{CP}(n)\, \cos\delta_n \sqrt{(\mathcal{B}(D^0 \to n)\, \mathcal{B}(\overline{D}^0 \to n)}$$

- δ_n is the strong phase difference between the $D^0 \to n$ and $\overline{D}^0 \to n$ amplitudes
- $\eta_{\text{CKM}} = (-1)^{n_s}$, where n_s is the number of s and $\bar{s}$ quarks in the final state.
- $CP|f\rangle = \eta_{CP}|\overline{f}\rangle$, well-defined as $|f\rangle$, $|\overline{f}\rangle$ in the same $SU(3)$ multiplet

$$y = \sum_a y_a\,, \qquad y_a = \eta_{CP}(a) \sum_{n \in a} \eta_{\text{CKM}}(n) \cos\delta_n \sqrt{\mathcal{B}(D^0 \to n)\, \mathcal{B}(\overline{D}^0 \to n)}$$

$$y_{\pi K} = \mathcal{B}(D^0 \to \pi^+\pi^-) + \mathcal{B}(D^0 \to K^+K^-) - 2\cos\delta_{K\pi} \sqrt{\mathcal{B}(D^0 \to K^-\pi^+)\, \mathcal{B}(D^0 \to K^+\pi^-)}$$

$$y_{\pi K} \approx (5.76 - 5.29 \cos\delta_{K\pi}) \times 10^{-3}$$

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$$y = \frac{1}{2\Gamma}\sum_n \rho_n \left[\langle D^0|\mathcal{H}_w|n\rangle\langle n|\mathcal{H}_w|\overline{D}^0\rangle + \langle \overline{D}^0|\mathcal{H}_w|n\rangle\langle n|\mathcal{H}_w|D^0\rangle\right]$$

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$$y_{\pi K} = \mathcal{B}(D^0 \to \pi^+\pi^-) + \mathcal{B}(D^0 \to K^+K^-) - 2\cos\delta_{K\pi}\sqrt{\mathcal{B}(D^0 \to K^-\pi^+)\,\mathcal{B}(D^0 \to K^+\pi^-)}$$

$$\frac{y_{\pi K}}{\mathcal{B}(h^+h^-)} \approx \frac{(5.22 - 4.18\cos\delta_{K\pi}) \times 10^{-3}}{4.3 \times 10^{-2}} \approx 2.3\% \quad \text{[PDG 2006/2007/2008]}$$

$SU(3)$ Breaking and D^0-$\overline{D}^0$ mixing

Falk, Grossman, Ligeti, and Petrov; Phys. Rev D65, 054034 (2002)

Final state representation		$y_{F,R}/s_1^2$	$y_{F,R}$ (%)
PP	8	-0.0038	-0.018
	27	-0.00071	-0.0034
PV	8_S	0.031	0.15
	8_A	0.032	0.15
	10	0.020	0.10
	$\overline{10}$	0.016	0.08
	27	0.040	0.19
$(VV)_{s\text{-wave}}$	8	-0.081	-0.39
	27	-0.061	-0.30
$(VV)_{p\text{-wave}}$	8	-0.10	-0.48
	27	-0.14	-0.70
$(VV)_{d\text{-wave}}$	8	0.51	2.5
	27	0.57	2.8

Values of $y_{F,R}$ for two-body final states. This represents the value which y would take if elements of F_R were the only channel open for D^0 decay.

Final state representation		$y_{F,R}/s_1^2$	$y_{F,R}$ (%)
$(3P)_{s\text{-wave}}$	8	-0.48	-2.3
	27	-0.11	-0.54
$(3P)_{p\text{-wave}}$	8	-1.13	-5.5
	27	-0.07	-0.36
$(3P)_{\text{form-factor}}$	8	-0.44	-2.1
	27	-0.13	-0.64
$4P$	8	3.3	16
	27	2.2	9.2
	27′	1.9	11

Values of $y_{F,R}$ for three- and four-body final states.

"On the basis of this analysis, in particular as applied to the $4P$ final state, we would conclude that y on the order of a percent would be completely natural. Anything an order of magnitude smaller would require significant cancellations which do not appear naturally in this framework. Cancellations would be expected only if they were enforced by the OPE, that is, if the charm quark were heavy enough that the "inclusive" approach were applicable. The hypothesis underlying the present analysis is that this is not the case."

Some Systematics & Detector Design

- Slow pion candidates used to tag the flavor of a neutral D can be *true*, *good false*, or *bad false*. Determining the ratio of *bad false to good false* accurately is difficult, so minimizing the number of *bad false* tags is important.
- A significant source of *bad false* candidates originates in $D^{*0} \rightarrow D^0 \pi^0$ with a photon converting to an e^+e^- pair.
- Specific ionization measurements (*dE/dx*) can be used effectively to remove these candidates, along with kaons and protons.
- Minimizing multiple scattering in the beam-pipe, minimizing the lever arm from the beam-spot to the first tracking measurement, and measuring the momentum as well as possible all contribute to the best possible resolution for Δm.

10 Years From Now ??

My guesstimates of measurement precision, assuming 100 fb^{-1} from LHCb and 50 ab^{-1} from Super*B*, in units of 10^{-4}

	x	y	y_{CP}	$(x')^2$	y'	x''	y''
$K\pi$				0.2	5		
h^+h^-			5				
$K_S\pi^+\pi^-$	5	5					
$K^+\pi^-\pi^0$						8	8
$\pi^-\pi^+\pi^0$	7	7					
$K_SK^+\pi^- + K_SK^-\pi^+$	7	7					

BES III should be able to measure cos δ $\pm$ 0.03

Super*B* and BES III will measure relevant branching fractions with 5% fractional precision, constraining Standard Model contributions to x & y.

Altogether, D^0-$\bar{D}^0$ mixing measurements, and measurements of CP-violation in mixing, will provide insights into physics beyond the SM that will complement direct observations made at the LHC.