Future of Lattice Calculations for *b* Physics

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Outline

Introduction

Required parameters

Target simulations

b physics

Conclusions

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Introduction

Will we be able to calculate hadronic parameters for *b*-physics with 1% or a few % precision by 2015?

Consider

- Required simulation parameters
- Scaling formulae and computational costs
- Requirements for *b*-physics

I rely heavily on:

- S Sharpe, *Weak Decays of Light Hadrons*, LQCD Present and Future, Orsay 2004
- V Lubicz, *CKM Fit and Lattice QCD*, SuperB IV, Monte Porzio Catone 2006
- C Sachrajda, *Prospects for Lattice Phenomenology*, LHCb Upgrade Workshop, Edinburgh 2007

Errors in lattice calculations

- Statistical

- Systematic

Errors in lattice calculations

- Statistical
  - Arise from Monte Carlo evaluation of functional integrals
  - Rule of thumb: about 100 *independent* configurations for $\sim 1\%$ statistical error
  - . . . but depends on quantity studied, lattice volume, exact formulation of LQCD used
- Systematic

Errors in lattice calculations

- Statistical
  - Arise from Monte Carlo evaluation of functional integrals
  - Rule of thumb: about 100 *independent* configurations for $\sim 1\%$ statistical error
  - ... but depends on quantity studied, lattice volume, exact formulation of LQCD used
- Systematic
  - Discretisation and continuum extrapolation ($a \neq 0$)
  - Light quarks: chiral extrapolation ($m_l \to m_{ud}$)
  - Finite volume ($L \neq \infty$)
  - Heavy quarks ($m_Q \to m_{c,b}$)
  - Renormalisation constants (matching lattice to continuum)

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Lattice spacing

Estimate from Sharpe, LQCD present and future, Orsay 2004

Assume using $O(a)$-improved action for observable $\mathcal{O}$

$$\mathcal{O}_{\mathrm{latt}} = \mathcal{O}_{\mathrm{phys}} \left[1 + c_2 (a\Lambda)^2 + c_n (a\Lambda)^n + \cdots \right]$$

- assume c_2, c_n are $O(1)$
- $n = 3, 4$ depending on action used
- $\Lambda \sim \Lambda_{\mathrm{QCD}}$ for light quarks
- $\Lambda \sim m_Q$ for heavy quarks Q (so more work needed to avoid lattice artefacts . . . see below)

Simulate at a_{min} and $\sqrt{2}a_{\mathrm{min}}$ and extrapolate linearly in a^2. Resulting error:

$$\frac{\Delta\mathcal{O}_{\mathrm{phys}}}{\mathcal{O}_{\mathrm{phys}}} \approx c_n (2^{n/2} - 2)(a_{\mathrm{min}}\Lambda)^n$$

Lattice spacing estimates

$$\frac{\Delta\mathcal{O}_{\text{phys}}}{\mathcal{O}_{\text{phys}}} \approx c_n(2^{n/2} - 2)(a_{\text{min}}\Lambda)^n$$

For 1% error (taking $c_n = 1$)

Λ	0.5 GeV	0.8 GeV	1.5 GeV	4.5 GeV
$a_{\text{min}}(n=3)$	0.091 fm	0.057 fm	0.030 fm	0.010 fm
$a_{\text{min}}(n=4)$	0.105 fm	0.066 fm	0.035 fm	0.012 fm

- Current lattice spacings $0.05\,\text{fm} \le a \le 0.13\,\text{fm}$
- OK for light quarks
- Daunting for charm
- Need effective theories for b

Minimum light quark mass

Estimate from ChPT:

$$\mathcal{O}_{\text{latt}} = \mathcal{O}_{\text{phys}} \left[1 + c_2 \left(\frac{m_\pi}{m_\rho} \right)^2 + c_4 \left(\frac{m_\pi}{m_\rho} \right)^4 + \cdots \right]$$

- Assume c_n are $O(1)$
- Simulate at $R_{\text{min}} = (m_\pi/m_\rho)_{\text{min}}$ and $\sqrt{2}R_{\text{min}}$ and extrapolate linearly in R^2
- Resulting error:

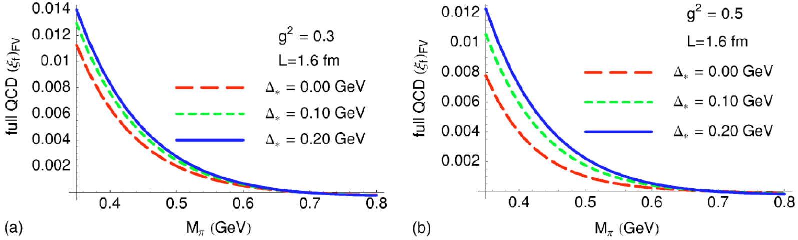
$$\frac{\Delta\mathcal{O}_{\text{phys}}}{\mathcal{O}_{\text{phys}}} \approx 2c_2 \left(\frac{m_\pi}{m_\rho} \right)^4_{\text{min}}$$

- For 1% error (taking $c_2 = 1$):

$$\left(\frac{m_\pi}{m_\rho} \right)_{\text{min}} \approx 0.27 \quad \text{or} \quad \frac{m_l}{m_s} \approx \frac{1}{11} \quad \text{or} \quad m_{\pi,\text{min}} \approx 210\,\text{MeV}$$

Finite volume: minimum box size

- FV effects matter when aiming for 1% precision
- Dominant effect from pion loops ⇒ estimate using ChPT
- Example: FV effects in f_{B_s}/f_{B_d} from HMChPT (Arndt, Lin, prd70 014503)



Finite volume effects

- For quantities without final state interactions

$$\frac{\Delta\mathcal{O}_{\text{phys}}}{\mathcal{O}_{\text{phys}}} \approx c e^{-m_\pi L}$$

  where c is $O(1)$, but depends on quantity calculated
- For 1% error (with $c = 1$)

$$m_\pi L \approx 4.6$$

- If $m_\pi = 200\,\text{MeV}$ then

$$L \approx 4.5\,\text{fm}$$

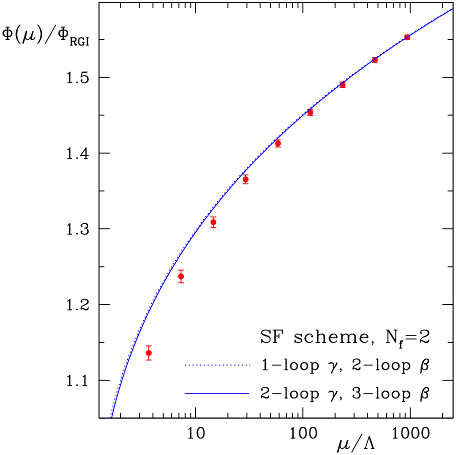
Heavy quarks

- From discussion above, a relativistic b quark would require $am_b \ll 1$, say

$$a \approx 0.01\,\mathrm{fm}$$

- This is *too small* even for Pflop computers
- Various approaches:
  - effective theories
  - interpolation between static limit and charm region
- ... see later

Renormalisation (Matching)



Della Morte, Fritzch, Heitger, JHEP 0702:079

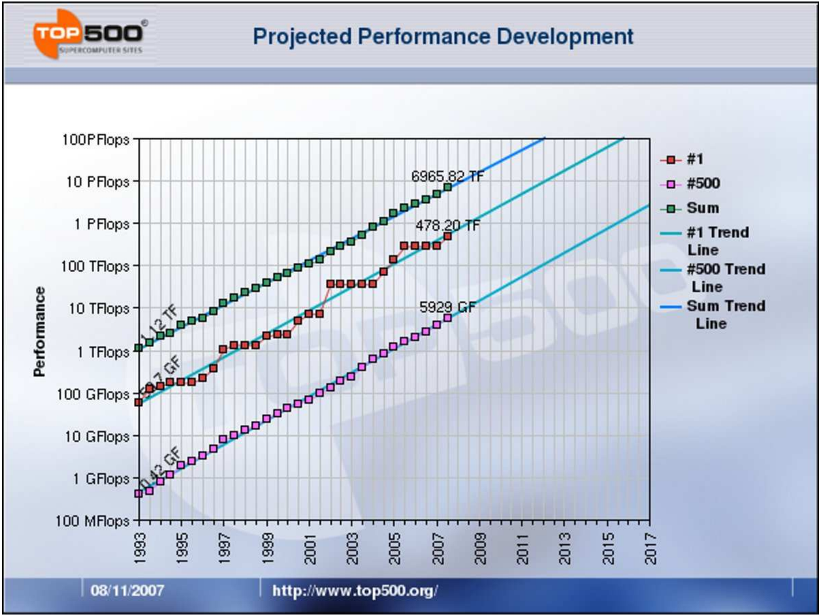
$$\mathcal{O}^{\rm R}(\mu) = Z(a\mu, g)\mathcal{O}^{\rm latt}(a)$$

- Nonperturbative (points) versus perturbative (curves) renormalisation of static-light axial current
- $N_f = 2$
- PT off by $\sim 5\%$ at hadronic scale
- Use NPR for 1% precision

Target simulations: aiming at 1% precision

	Light quarks	Charm quarks
N_{conf}	120	120
a	1/20 fm	1/30 fm
a^{-1}	$\approx 4\,\mathrm{GeV}$	$\approx 6\,\mathrm{GeV}$
m_l/m_s	1/12	1/12
m_π	200 MeV	200 MeV
L	4.5 fm	4.5 fm
Vol	$90^3 \times 180$	$140^3 \times 280$

- Tough for charm; *b* not directly simulated on full-size lattice
- Are such simulations feasible? Compare computer power to estimated computational cost


TOP500
SUPERCOMPUTER SITES
Projected Performance Development
Performance
100PFlops
10 PFlops
1 PFlops
100 TFlops
10 TFlops
1 TFlops
100 GFlops
10 GFlops
1 GFlops
100 MFlops
1993
1995
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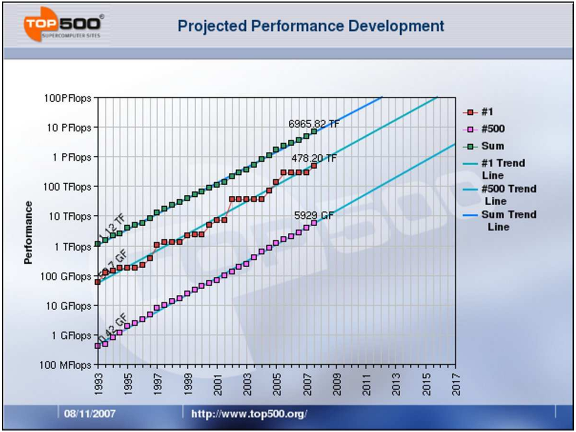
2007
2009
2011
2013
2015
2017
6965.82 TF
478.20 TF
5929 GF
1.12 TF
0.42 GF
#1
#500
Sum
#1 Trend Line
#500 Trend Line
Sum Trend Line
08/11/2007
http://www.top500.org/

Computer power



LQCD

- 1–10 Tflop/s today
- 1–10 Pflop/s 2015

Varieties of fermions

Wilson **Staggered** **Ginsparg-Wilson**

Varieties of fermions

Wilson
- standard
- $O(a)$-improved
- twisted mass

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- first to reach light masses: $m_l/m_s \sim 1/8$
- "ugly" (Sharpe, hep-lat/0610094)

Ginsparg-Wilson

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Ginsparg-Wilson

- Staggered ⇒ 4 *tastes* per flavour
- Reduced to one by 4th root of quark determinant
- Rooted staggered fermions unphysical for $a \neq 0$, but go over to single-taste theory in limit $a \to 0$.
- rSχPT: complicated fits with unphysical effects included in fit

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Ginsparg-Wilson

- domain wall
- overlap
- good chiral properties
- 10–30 price hike

Algorithmic progress

Tremendous progress in C21.

- RHMC (Clark–Kennedy, NPBPS129 850, PRL98 051601)
- Mass preconditioning (Hasenbusch, PLB519 177; Urbach et al, CPC174 87)
- Domain-decomposition (Del Debbio et al, JHEP02 056)

Algorithmic progress

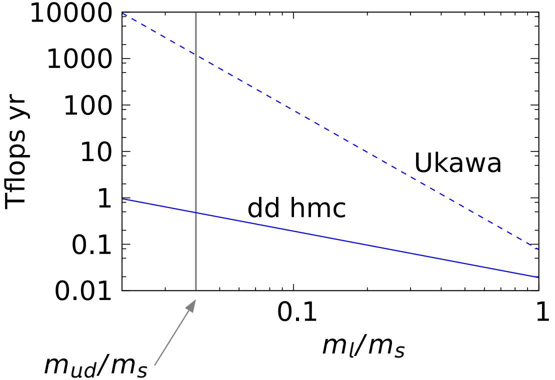
Compare 100 configurations of $N_f = 2$, $O(a)$-improved Wilson fermions:

- 2001 (Ukawa, Lattice2001)

$$5 \left[\frac{0.2}{m_l/m_s}\right]^3 \left[\frac{L}{3\,\mathrm{fm}}\right]^5 \left[\frac{0.1\,\mathrm{fm}}{a}\right]^7 \text{ TflopsYr}$$

- 2006 (Del Debbio et al, JHEP02 056): DD-HMC

$$0.05 \left[\frac{0.2}{m_l/m_s}\right] \left[\frac{L}{3\,\mathrm{fm}}\right]^5 \left[\frac{0.1\,\mathrm{fm}}{a}\right]^6 \text{ TflopsYr}$$



100 conf

$L = 2.5\,\mathrm{fm}$

$a = 0.08\,\mathrm{fm}$

$V = 32^3 \times 64$

Cost estimates: Wilson fermions

	Light quarks	Charm quarks
N_{conf}	120	120
a	1/20 fm	1/30 fm
a^{-1}	$\approx 4\,\mathrm{GeV}$	$\approx 6\,\mathrm{GeV}$
m_l/m_s	1/12	1/12
m_π	200 MeV	200 MeV
L	4.5 fm	4.5 fm
Vol	$90^3 \times 180$	$140^3 \times 280$
Wilson	0.07 Pflops yr	0.9 Pflops yr

- Overhead for $N_f = 2 + 1$ and generating extra ensembles at larger a and larger m_l is a factor of about 3
- Bigger overhead for GW simulations (with good chiral symmetry)

Cost estimate: DWF fermions

DWF scaling formula (Christ and Jung, Lattice 2007)

$$\text{Cost} \propto \left[\frac{L}{\text{fm}}\right]^5 \left[\frac{\text{MeV}}{m_\pi}\right] \left[\frac{\text{fm}}{a}\right]^6 \times \left\{ C_0 + C_1 \left[\frac{\text{MeV}}{m_K}\right]^2 \left[\frac{\text{fm}}{a}\right] + C_2 \left[\frac{\text{MeV}}{m_\pi}\right]^2 \left[\frac{a}{\text{fm}}\right]^2 \right\}$$

- About $1.5\,\text{Pflops yr}$ for the light quark target simulation
- May not need such small a ($0.05\,\text{fm}$) for DWF
- Physics projects may demand larger volumes? ($L > 4.5\,\text{fm}$)
- RBC–UKQCD able to do this around 2011?

b physics on the lattice

Simulating a relativistic b-quark with 1% errors needs $a \sim 0.01\,\mathrm{fm}$

- Cost scales as a^{-6} or a^{-7}
- Prohibitive even for Pflops computers if you want a big ($L \approx 4.5\,\mathrm{fm}$) lattice as well

Charm physics *is* feasible with Wilson fermions

- For $a = 0.033\,\mathrm{fm}$, cost for 120 configurations $\sim 0.9\,\mathrm{Pflops\,yr}$

Lattice *b*-physics: complementary approaches

- Simulate relativistic quarks in charm region and extrapolate to *b*
- Effective theories
  - HQET: substantial progress in nonperturbative renormalisation, use of static-link fattening and inclusion of $O(\Lambda_{\mathrm{QCD}}/m_b)$ corrections
  - NRQCD or Fermilab/Tsukuba (RHQ) actions
- Finite-volume and step-scaling approach of Rome-II group

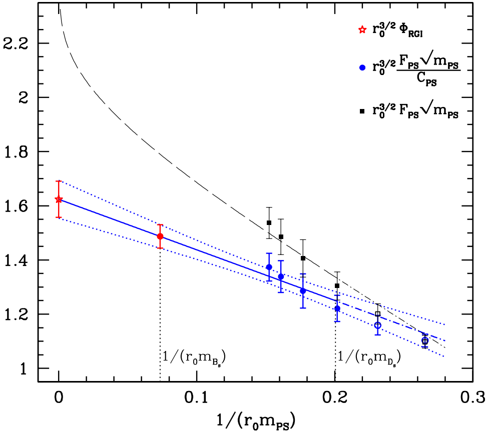
$$\mathcal{O}(L_\infty) = \mathcal{O}(L_0)\frac{\mathcal{O}(L_1)}{\mathcal{O}(L_0)}\cdots\frac{\mathcal{O}(L_N)}{\mathcal{O}(L_{N-1})}$$

  L_0 small enough to allow $a \approx 0.01\,\mathrm{fm}$ and $L_N \sim L_\infty$ (last factor is 1 to required precision)
- Step-scaling also used for nonperturbative renormalisation of HQET (ALPHA) and RHQ (Christ & Lin, prd76 074505/6)

Current status: interpolation

- ALPHA have implemented interpolation between static results and relativistic charm-scale results
- ALPHA and Rome-II have combined static and step-scaling results
- Both reach 3% precision for f_{B_s}
- . . . but still in quenched approximation and consider f_{B_s} so no chiral extrapolation

Interpolation: static and relativistic

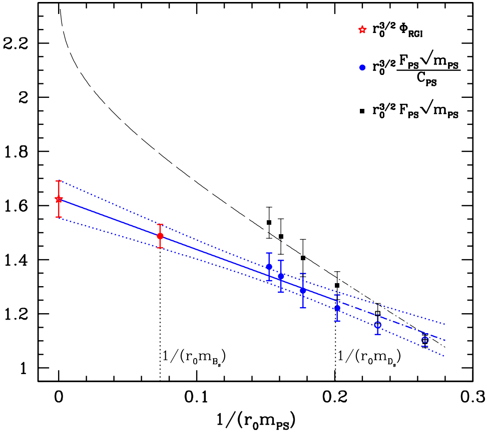


Della Morte et al, arXiv:0710.2201

$f_{B_s} = 193(6)\,\mathrm{MeV}$

- $a \to 0$ before $1/m_{\mathrm{PS}}$ interpolation
- still quenched
- no chiral extrapolation

Interpolation: static and relativistic/step-scaling



Della Morte et al, arXiv:0710.2201

Guazzini et al, arXiv:0710.2229

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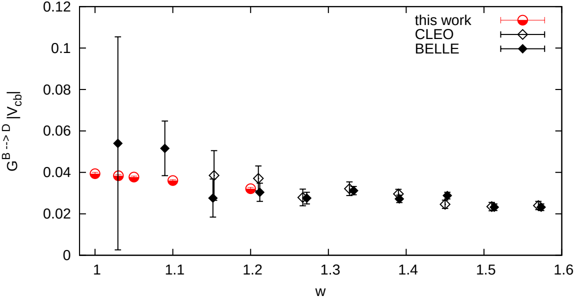
Heavy-to-heavy semileptonic decay: $B \to Dl\nu$



$G^{B\to D}(\omega)|V_{cb}|$

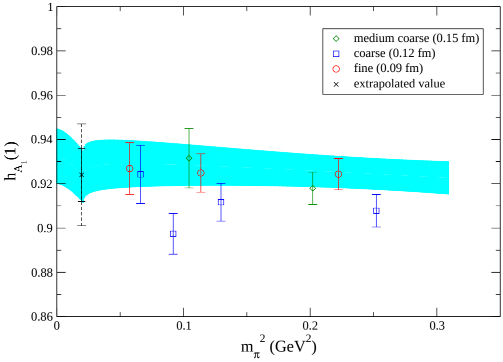
de Divitiis et al, PLB655 45

- Rome-II step-scaling method plus twisted BCs
- Lattice data normalized to experiment at $\omega = 1.2$
- 2% error on $G(\omega{=}1)$ ... quenched

Heavy-to-heavy semileptonic decay: $B \to D^* l\nu$

Extract h_{A_1} directly from double ratio:

$$|h_{A_1}(1)|^2 = \frac{\langle D^*|\bar{c}\gamma_j\gamma_5 b|\bar{B}\rangle\langle\bar{B}|\bar{b}\gamma_j\gamma_5 c|D^*\rangle}{\langle D^*|\bar{c}\gamma_4 c|D^*\rangle\langle\bar{B}|\bar{b}\gamma_4 b|\bar{B}\rangle}$$



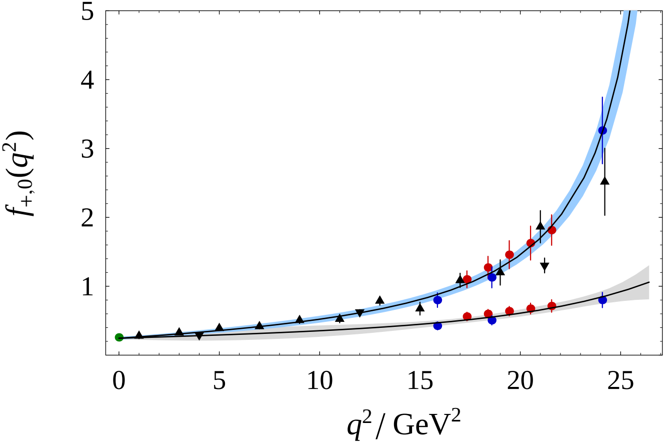
Plot: $h_{A_1}(1)$ vs m_π^2

- $2+1$ improved staggered ⇒ rSχPT fit
- Fermilab heavy quarks
- Quote 2.3% error

Laiho, arXiv:0710.1111

Current status: $B \to \pi$ semileptonic decays

- Results from Fermilab and HPQCD using different effective theories
  - Fermilab: Fermilab action (not final . . .)
  - HPQCD: NRQCD (prd73 074502, prd75 119906(E))
- Results have come into agreement
- . . . but, based on same gauge field ensembles
- HPQCD: biggest errors from chiral extrapolation and perturbative matching
- $\sim 14\%$ error on form factors in $q^2 \geq 16\,\mathrm{GeV}^2$
- Need confirmation from other approaches



$$|V_{ub}| = 3.47(29)(03) \times 10^{-3}$$
$$f_+(0) = 0.245(23)$$
$$|V_{ub}|f_+(0) = 8.5(8) \times 10^{-4}$$

JMF & Nieves, prd76 031302

b-physics prognosis

- *Best* results likely from combining extrapolation from $m_Q \approx m_c$ with effective theory results (including $\Lambda_{\mathrm{QCD}}/m_b$ corrections)
- Few % precision requires nonperturbative renormalisation: this is being done for HQET
- Medium term: look for agreement between different approaches (HQET, NRQCD, Fermilab/Tsukuba) and study theoretical foundations
- Although quenched approximation has been banished from light quark physics, some heavy quark analysis still being developed using quenched ensembles: redo unquenched once methods established

Conclusions

- Access to Pflops computers with current techniques and knowledge should allow few % precision in *b*-physics
- Further theoretical and technical advances will likely improve precision further
- Need all hadrons strongly stable (so not $B \to \rho$ decays for now)
- For *b*-hadron decays to two-hadron (or higher) states we need new ideas before we can formulate a numerical approach to evaluating the amplitudes