Model independent analysis of B meson light-cone wavefunction

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Beneke et al. ('99)
Bauer, Pirjol, Stewart ('01)
Keum, Li, Sanda ('01)

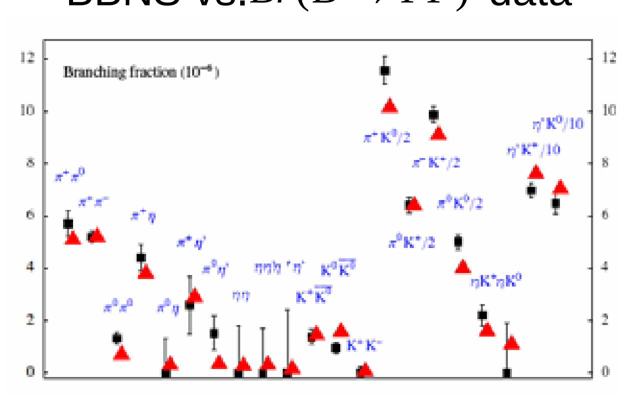
Introduction

QCD Factorization for Exclusive B decays: $B \to \pi\pi,\ \pi l \nu,\ \gamma l \nu \ldots$

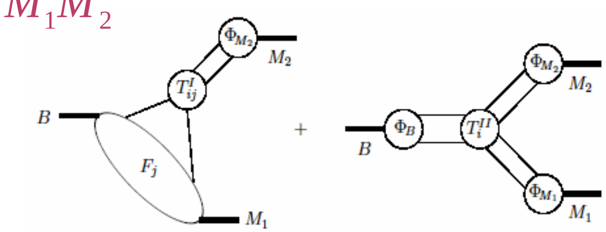
→ systematic framework to calculate QCD effects in the $m_b \to \infty$ limit

- successful to describe the hadronic B decays.

BBNS vs. $Br(B \to PP)$ data



$B \to M_1 M_2$

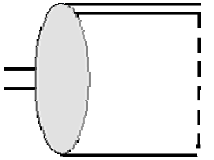


Beneke (Beauty06)

- In some cases, the ignorance of B meson wavefunction gives a major theoretical uncertainty. ex. hard spectator interactions in $B \to \pi^0 \pi^0$

$$\lambda_B^{-1}(\mu) = \int \frac{d\omega}{\omega} \phi_B(\omega,\mu) \quad \sigma_B \lambda_B^{-1}(\mu) = \int \frac{d\omega}{\omega} \phi_B(\omega,\mu) \ln\left(\frac{\omega}{\mu}\right)$$ etc.

B meson LCWF



- B meson light-cone wavefunction in HQET

$$\tilde{\phi}_B(t,\mu) = \langle 0 | \left[\bar{q}(tn) \mathcal{P} \exp\left(\int_0^t d\lambda n_\mu A^\mu(\lambda n) \right) \not{n} \gamma_5 h_v(0) \right]_\mu | \bar{B}(v) \rangle$$

$n^\mu = (1,0,0,-1)$ light-cone vector: $p^\mu = m_B v^\mu \ (v^2 = 1)$

HQET field: $b(x) = e^{-im_Q v\cdot x} h_v(x) + \mathcal{O}\left(1/m_Q\right), \ \not{v} h_v(x) = h_v(x)$

In momentum space $\phi_B(\omega,\mu^2) = \int \frac{dt}{2\pi} e^{i\omega t} \tilde{\phi}_B(t,\mu^2)$

$k^+ = \omega v^+$ momentum of light quark

- UV structure (radiative tail)

$$\phi_B(\omega) \sim -iF\alpha_s \frac{\log(\omega/\mu)}{\omega} \Longrightarrow \int_0^\infty d\omega \omega^j \phi_B(\omega) = -\infty \ (j \geq 0)$$

Radiative corrections generate a hard negative tail. ↔ pion LCWF

IR structure

Kodaira, Tanaka, Qiao, HK ('01)

HQ symmetry $\not{v} h_v = h_v$ + E.O.M. $\bar{q}\overleftarrow{\not{D}} = v \cdot \overrightarrow{D} h_v = 0$

$$\rightarrow \quad \phi_B(\omega) = \phi_B^{(WW)}(\omega) + \phi_B^{(g)}(\omega)$$

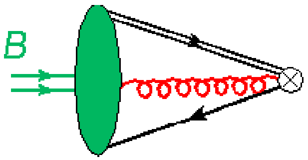
$\bar{\Lambda} = m_B - m_b$

$\langle 0|\bar{q}\ \not{n}\gamma_5 h_v|\bar{B}\rangle = iF$

"Wandzura-Wilczek" part $\phi_B^{(WW)}(\omega) = iF \dfrac{\omega}{2\bar{\Lambda}^2}\theta(2\bar{\Lambda} - \omega)$

$$\leftrightarrow \ \left\langle 0 \left| q(0)(v \cdot D)^n h_v \right| \bar{B}(v) \right\rangle$$

$$\phi_B^{(g)} \sim \ \left\langle 0 \left| \bar{q} G h_v \right| \bar{B}(v) \right\rangle$$

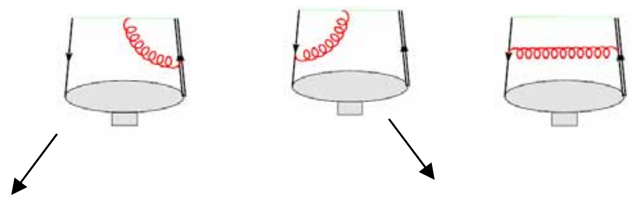


— "twist = dimension - spin" is not a good quantum number

— Contributions from higher dim. Operators in the IR region.

Radiative corrections

Lange & Neubert ('03), Braun et al. (03), Li & Liao ('04), Lee & Neubert ('05)

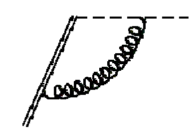


$$L = \log(it\mu), \quad \mu = e^{\gamma_E}\mu_{\overline{\rm MS}}$$

$$\tilde{\phi}_B^{\text{1-loop}}(t,\mu)$$
$$= \frac{\alpha_s C_F}{2\pi}\int_0^1 d\xi \left[\left\{-\left(\frac{1}{2\varepsilon_{UV}^2}+\frac{L}{\varepsilon_{UV}}+L^2+\frac{5\pi^2}{24}\right)\delta(1-\xi)+\left(\frac{1}{\varepsilon_{UV}}-\frac{1}{\varepsilon_{IR}}\right)\left[\frac{\xi}{1-\xi}\right]_+\right\}\langle 0|\bar{q}(\xi t n)\not{n}\gamma_5 h_v(0)|\bar{B}(v)\rangle\right.$$
$$-\left(\frac{1}{2\varepsilon_{IR}}+L\right)\langle 0|\bar{q}(\xi t n)\not{n}\gamma_5 h_v(0)|\bar{B}(v)\rangle - t\left(\frac{1}{\varepsilon_{IR}}+2L-1-\xi\right)\langle 0|\bar{q}(\xi t n)v\cdot\overleftarrow{D}\not{n}\gamma_5 h_v(0)|\bar{B}(v)\rangle$$
$$\left. + O(t^2)\ (\text{IR poles, higher-dim ops.})\right]$$

- cusp singularity → radiative tail $\log^2(it\mu)$
- $\log(it\mu)$ non-analytic at $t = 0$
- UV & IR structures are different!

CUSP



$$h_v(0) = P\exp\left(ig\int_{-\infty}^{0} ds\, v_\mu A^\mu(sv)\right)h_v(-\infty v)$$

RG evolution

$$\mu\frac{d}{d\mu}\tilde{\phi}_B(t,\mu) = \frac{\alpha_s}{2\pi}\int_0^1 d\xi\left[-\left(2L-\frac{1}{2}\right)\delta(1-\xi)+\left(\frac{\xi}{1-\xi}\right)_+\right]\tilde{\phi}_B(\xi t,\mu)$$

Operator product expansion

$\log^2(it\mu),\ \log(it\mu)$ $\quad \mu \Leftrightarrow 1/t$ correlated

- $1/t$ gives a natural scale to separate UV & IR behaviors $\Rightarrow$ OPE
- B meson LCWF in terms of HQET parameters

UV

$\mu_F = \sqrt{m_b \Lambda}$ — $\dfrac{1}{\varepsilon_{UV}^2},\ \dfrac{1}{\varepsilon_{UV}}$

RG evolution for LCWF

$$\tilde{\phi}_B(t,\mu_F) = \widehat{U}(\mu_F,\mu) \otimes \tilde{\phi}_B(t,\mu)$$

$1/t$

OPE at $\mu \le \dfrac{1}{t}$

$$\tilde{\phi}_B(t,\mu) = \sum_i \tilde{C}_i(t,\mu) \langle 0 | O_i(\mu) | \bar{B}(v) \rangle$$

μ

expansion in $t\Lambda$

IR $\quad \Lambda$ — $\dfrac{1}{\varepsilon_{IR}}$ with higher dim. ops.

OPE at NLO

- Lee & Neubert PRD72('05)094028

$$\tilde{\phi}_B(t,\mu,\Lambda_{\rm UV}) = \sum_i \tilde{C}_i(t,\mu,\Lambda_{\rm UV})\langle 0|O_i(\mu)|\bar{B}(v)\rangle$$

cut-off scheme, up to dim-4 ops. $\bar{q}\Gamma h_v \quad \bar{q}D\Gamma h_v$

- This work

$$\tilde{\phi}_B(t,\mu) = \sum_i \tilde{C}_i(t,\mu)\langle 0|O_i(\mu)|\bar{B}(v)\rangle$$

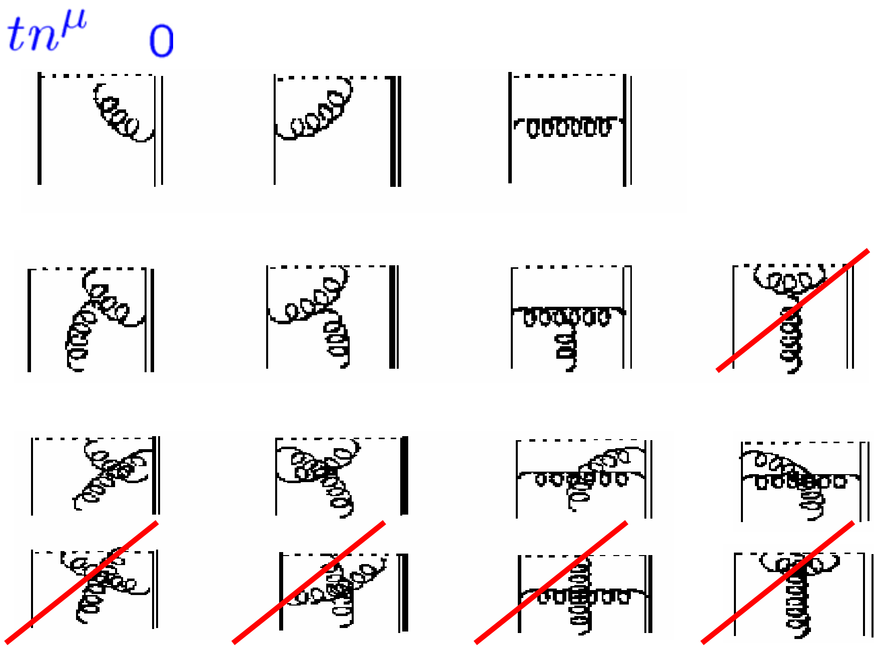
MS-bar scheme, up to dim-5 ops. $\bar{q}\Gamma h_v \quad \bar{q}D\Gamma h_v$

$+ \quad \{\bar{q}DD\Gamma h_v,\ \bar{q}G\Gamma h_v\}$

Calculation

- 1-loop matching of the non-local operator for B meson LCDA with non-local ops. up to dim.5

$$\bar{q}(tn)\mathrm{P}\exp\left(ig\int_0^t d\lambda n_\mu A^\mu(\lambda n)\right)\not{n}\gamma_5 h_v(0) \leftrightarrow \bar{q}\Gamma h_v,\ \ \bar{q}D\Gamma h_v,\ \ \left\{\bar{q}DD\Gamma h_v, \bar{q}G\Gamma h_v\right\}$$



many dim. 5 ops.

→ calculation in x-space keeping gauge invariance explicitly.

- Background field method

$$A_\mu = A_\mu^{cl} + A_\mu^{q}$$

- Fock-Schwinger gauge for A_μ^{cl}

$$x^\mu A_\mu^{(C)}(x) = 0$$

$$\Rightarrow \quad A_\mu^{(C)}(x) = \int_0^1 du u x^\rho G_{\rho\mu}^{(C)}(ux)$$

→ decouple from Wilson line

OPE up to dim.5 (NLO in MS-bar scheme)

$L = \log(i\mu t)$ $\mu = e^{\gamma_E}\mu_{\overline{\rm MS}}$

$$
\begin{aligned}
&\bar{q}(tn)\mathcal{P}\exp\left(\int_0^t d\lambda n_\mu A^\mu(\lambda n)\right)\not{n}\gamma_5 h_v(0) \\
&= \bar{q}\not{n}\gamma_5 h_v\left[1+\frac{\alpha_s}{4\pi}C_F\left(-2L^2-2L-\frac{5}{12}\pi^2\right)\right] && \text{dim.3}\\
&+(-it)\left\{\bar{q}(in\cdot\overleftarrow{D})\not{n}\gamma_5 h_v\left[1+\frac{\alpha_s}{4\pi}C_F\left(-2L^2-L-\frac{5}{12}\pi^2\right)\right]\right. && \text{dim.4}\\
&\left.+\bar{q}(iv\cdot\overleftarrow{D})\not{n}\gamma_5 h_v\left[\frac{\alpha_s}{4\pi}C_F(-4L+3)\right]\right\}\\
&+\frac{(-it)^2}{2}\left\{\bar{q}(in\cdot\overleftarrow{D})^2\not{n}\gamma_5 h_v\left[1+\frac{\alpha_s}{4\pi}C_F\left(-2L^2-\frac{2}{3}L-\frac{5}{12}\pi^2\right)\right]\right. && \text{dim.5}\\
&+\bar{q}(iv\cdot\overleftarrow{D})(in\cdot\overleftarrow{D})\not{n}\gamma_5 h_v\left[\frac{\alpha_s}{4\pi}C_F\left(-4L+\frac{10}{3}\right)\right]\\
&+\bar{q}(iv\cdot\overleftarrow{D})^2\not{n}\gamma_5 h_v\left[\frac{\alpha_s}{4\pi}C_F\left(-4L+\frac{10}{3}\right)\right]\\
&+\bar{q}igG_{\mu\nu}v^\mu n^\nu\not{n}\gamma_5 h_v\left[\frac{\alpha_s}{4\pi}\left\{C_F\left(-4L+\frac{10}{3}\right)+C_G\left(7L-\frac{13}{2}\right)\right\}\right]\\
&+\bar{q}igG_{\mu\nu}\gamma^\mu n^\nu\bar{\not{n}}\gamma_5 h_v\left[\frac{\alpha_s}{4\pi}\left\{C_F\left(-\frac{4}{3}L+\frac{4}{3}\right)+C_G(L-1)\right\}\right]\\
&+\bar{q}igG_{\mu\nu}\gamma^\mu v^\nu\bar{\not{n}}\gamma_5 h_v\left[\frac{\alpha_s}{4\pi}\left\{C_F\left(-\frac{2}{3}L+\frac{2}{3}\right)+C_G(L-1)\right\}\right]\\
&\left.+\bar{q}gG_{\mu\nu}\sigma^{\mu\nu}\not{n}\gamma_5 h_v\left[\frac{\alpha_s}{4\pi}\left\{C_F\left(-\frac{L}{3}+\frac{1}{3}\right)+C_G\left(\frac{L}{4}-\frac{1}{4}\right)\right\}\right]\right\}
\end{aligned}
$$

Matrix elements

dim.3 $\langle 0|\bar{q}\not{v}\gamma_5 h_v|\bar{B}(v)\rangle = iF(\mu)$ ← decay constant

dim.4 $\langle 0|\bar{q}(iv\cdot \overleftarrow{D})\not{v}\gamma_5 h_v|\bar{B}(v)\rangle = iv\cdot\partial\langle 0|\bar{q}\not{v}\gamma_5 h_v|\bar{B}(v)\rangle = iF(\mu)\bar{\Lambda}$

$\langle 0|\bar{q}(in\cdot \overleftarrow{D})\not{n}\gamma_5 h_v|\bar{B}(v)\rangle = iF(\mu)\frac{4}{3}\bar{\Lambda}$ $\bar{\Lambda} = m_B - m_b$

dim.5 (covariant tensor formalism)

$$\langle 0|\bar{q}\,\overleftarrow{D}_\mu\overleftarrow{D}_\nu\,\Gamma h_v|\bar{B}(v)\rangle = \frac{iF(\mu)}{2}\mathrm{Tr}\left[\gamma_5\Gamma\frac{1+\not{v}}{2}\{c_1 v_\mu v_\nu + c_2 g_{\mu\nu} + c_3(\gamma_\mu v_\nu + \gamma_\nu v_\mu) + c_4(\gamma_\mu v_\nu - \gamma_\nu v_\mu) + c_5 i\sigma_{\mu\nu}\}\right]$$

$$\begin{aligned} c_1 &= 2\bar{\Lambda}^2 + \frac{2}{3}\lambda_E^2 + \frac{1}{3}\lambda_H^2 \\ c_2 &= -\frac{\bar{\Lambda}^2}{3} - \frac{\lambda_E^2}{3} - \frac{\lambda_H^2}{3} \\ c_3 &= -\frac{\bar{\Lambda}^2}{3} - \frac{\lambda_E^2}{6} \\ c_4 &= \frac{1}{6}(\lambda_E^2 - \lambda_H^2) \\ c_5 &= \frac{\lambda_H^2}{6} \end{aligned}$$

"Chromo-electronic" $v^\mu = (1,\mathbf{0})$

$$\langle 0|\bar{q}\alpha\cdot g\mathbf{E}\gamma_5 h_v|\bar{B}(v)\rangle = F(\mu)\lambda_E^2(\mu)$$

"Chromo-magnetic"

$$\langle 0|\bar{q}\sigma\cdot g\mathbf{H}\gamma_5 h_v|\bar{B}(v)\rangle = iF(\mu)\lambda_H^2(\mu)$$

B meosn LCWF from OPE

$$\frac{\tilde{\phi}_B(t,\mu)}{iF(\mu)} = 1 - \frac{\alpha_s(\mu)}{4\pi}C_F\left(2L^2 + 2L + \frac{5}{12}\pi^2\right) \quad \text{dim.3}$$
$$-it\frac{4\bar{\Lambda}}{3}\left[1 - \frac{\alpha_s(\mu)}{4\pi}C_F\left(2L^2 + 4L - \frac{9}{4} + \frac{5}{12}\pi^2\right)\right] \quad \text{dim.4}$$
$$-t^2\bar{\Lambda}^2\left[1 - \frac{\alpha_s(\mu)}{4\pi}C_F\left(2L^2 + \frac{16}{3}L - \frac{35}{9} + \frac{5}{12}\pi^2\right)\right] \quad \text{dim.5}$$
$$-t^2\frac{\lambda_E^2(\mu)}{3}\left[1 - \frac{\alpha_s(\mu)}{4\pi}\left\{C_F\left(2L^2 + 2L - \frac{2}{3} + \frac{5}{12}\pi^2\right) + C_G\left(\frac{3}{4}L - \frac{1}{2}\right)\right\}\right]$$
$$-t^2\frac{\lambda_H^2(\mu)}{6}\left[1 - \frac{\alpha_s(\mu)}{4\pi}\left\{C_F\left(2L^2 + \frac{2}{3} + \frac{5}{12}\pi^2\right) + C_G\left(\frac{1}{2}L - \frac{1}{2}\right)\right\}\right]$$

- Dim.3&4 terms reproduce the results in cut-off scheme by Lee & Neubert ('05)
- Expressed in terms of 3 HQET parameters: $\bar{\Lambda}, \lambda_E^2, \lambda_H^2$
- $\bar{\Lambda}$ in on-shell scheme suffers IR renormalon ambiguities.

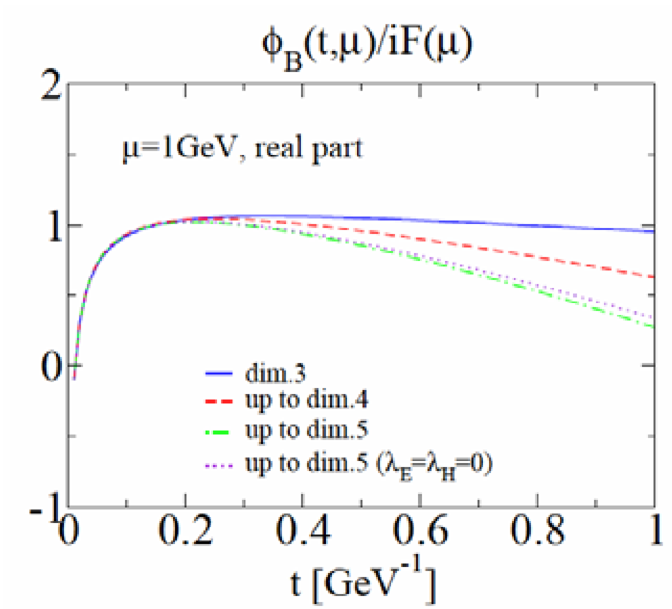
  → "distribution amplitude" scheme

$$\bar{\Lambda} = \bar{\Lambda}_{\rm DA}(\mu_f,\mu)\left[1 - \frac{\alpha_s C_F}{4\pi}\left(6\ln\frac{\mu_F}{\mu} - \frac{7}{4}\right)\right] + \mu_f\frac{\alpha_s C_F}{4\pi}\left(3\ln\frac{\mu_f}{\mu} - \frac{9}{2}\right) + \cdots$$

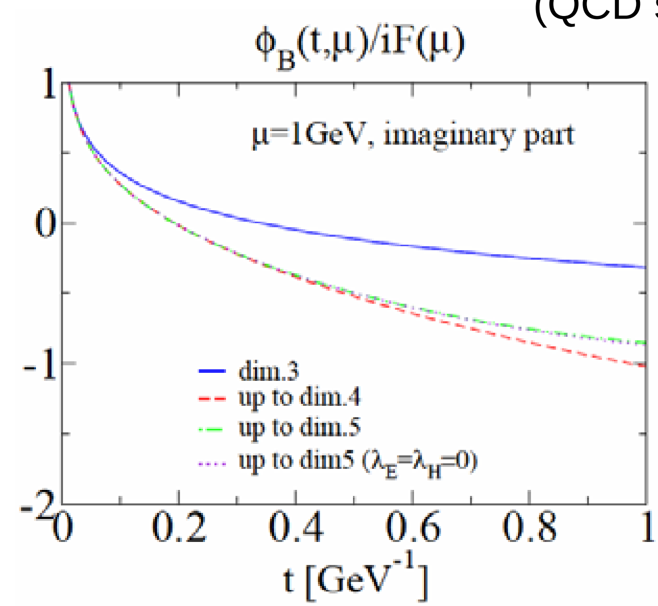
LCWF from OPE at $\mu = 1\mathrm{GeV}$

Inputs: $\bar{\Lambda}_{DA}(\mu_F, \mu) = 0.519\mathrm{GeV}$ at $\mu_F = \mu = 1\mathrm{GeV}$ Lee,Neubert ('05)

$\lambda_E^2 = 0.11\mathrm{GeV}^2$, $\lambda_H^2 = 0.18\mathrm{GeV}^2$ Grozin,Neubert ('97) (QCD sum rule)



real part



imaginary part

- Contributions from dim. 5 operators are important.
- Effects from λ_E, λ_H are very small.
- Evolution effects must be included.

Summary

- Knowledge of B meson LCWF is important to reduce the theory error for exclusive B decays.

- Radiative correction to B meson LCWF $\tilde{\phi}_B(t,\mu)$

  — different UV & IR structures

  — $\log^2(it\mu),\ \log(\mathrm{it}\mu)$ $\rightarrow$ $\mu \Leftrightarrow 1/t$

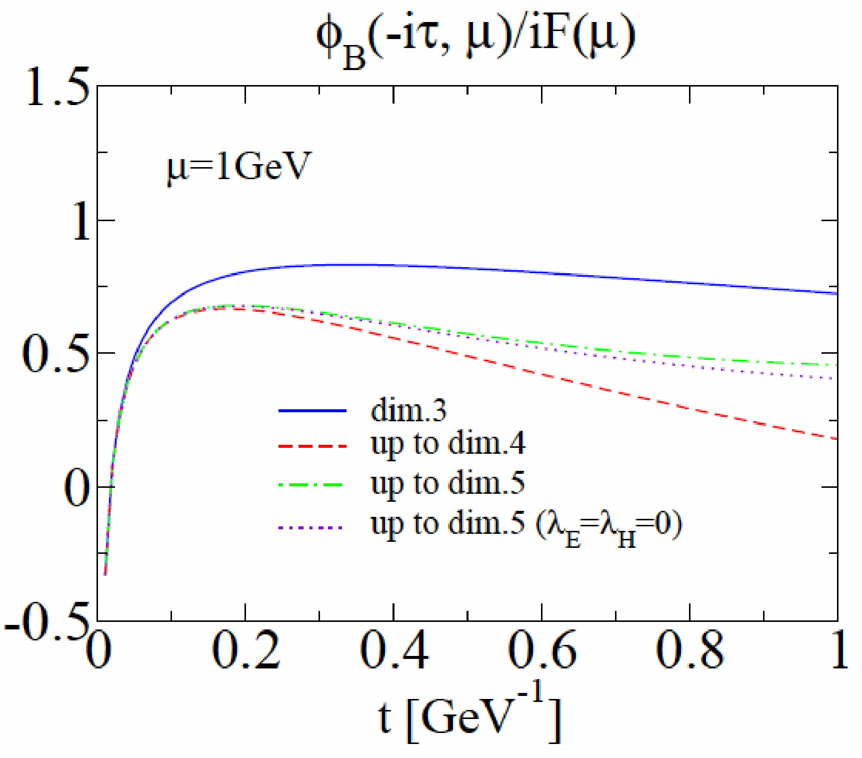
- OPE for B meson LCWF
  — up to dim.5
  — NLO corrections to Wilson coefficient

- $\tilde{\phi}_B(t,\mu)$ expressed in terms of 3 HQET parameters $\overline{\Lambda},\ \lambda_E,\ \lambda_H$ at $\mu \sim 1\mathrm{GeV}$

- Model-independent study including RG evolution is underway.

- Similar analysis for shape function is possible.

LCWF with analytic continuation $t \to -i\tau$



$$\lambda_B^{-1}(\mu) = \int_0^\infty d\tau \tilde{\phi}_B(-i\tau, \mu)$$