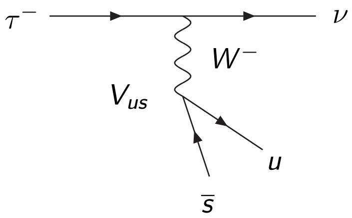
CP violation of $\tau \to \nu K \pi^0(\eta)$

T. Morozumi(Hiroshima)
Collaboration, Kang Young Lee(Korea),
Daiji Kimura(Hiroshima), Keita Nakagawa(Hiroshima)

Jan. 26 /2008, BNM-III 2008, Atami

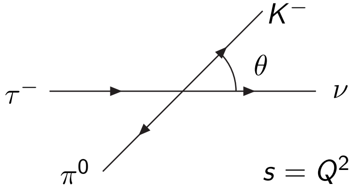
- Belle studied $\tau^{\pm} \to K_s \pi^{\pm} \nu$ (hep-ex 0706.2231, Physics Lett.B.).
- Preliminary measurement of $\tau^{\pm} \to K^{\pm} \eta \nu$ is reported.(By K. Hayasaka, YITP workshop at Kyoto)
- We study CP violation of $\tau^{\pm} \to K^{\pm} \pi^0 (\eta) \nu$



CP violation measurement $\longrightarrow$ unpolarized case.

Forward and Backward Asymmetry

$$\frac{dA_{\rm FB}}{d\sqrt{s}} = \frac{d\Gamma}{d\sqrt{s}}\Big|_{\cos\theta>0} - \frac{d\Gamma}{d\sqrt{s}}\Big|_{\cos\theta<0} = -\frac{G_F^2|V_{us}|^2}{2^5\pi^3}\frac{(m_\tau^2-s)^2}{m_\tau^3}\frac{m_\tau^2}{\sqrt{s}}I(s)^2{\rm Re}F_V F_S^*$$



$$s = Q^2 = (p_K + p_\pi)^2$$

Vector F_V and scalar F_S form factors

$$\langle K^-(p_k)\pi^0(p_\pi)|\bar{s}\gamma^\mu u|0\rangle = \left(g^{\mu\nu} - \frac{Q^\mu Q^\nu}{Q^2}\right) q_\nu F_V(Q^2) + Q^\mu F_S(Q^2)$$

differential rate

$$\frac{d^2\Gamma}{d\sqrt{s}d\cos\theta} = \frac{G_F^2|V_{us}|^2}{2^5\pi^3}\frac{(m_\tau^2 - s)^2}{m_\tau^3} I(s)$$
$$\Bigg((\frac{m_\tau^2}{s}\cos^2\theta + \sin^2\theta) I(s)^2 |F_V(s)|^2 + \frac{m_\tau^2}{4}|F_s(s)|^2$$
$$- \frac{m_\tau^2}{\sqrt{s}} I(s)\cos\theta \mathrm{Re}(F_V F_s^*) \Bigg)$$

- Forward and Backward Asymmetry $\rightarrow$ Inteference bet. $L = 0$ and $L = 1$ states of $K\pi$ system.
- F_V and F_S have their own strong phases. (Non-perturbative input).
- By taking the direct CP asymmetry (CP conjugate process) $\overline{A} = \overline{A}(\tau^+ \rightarrow \overline{\nu} K^+ \pi^0)$, we may have

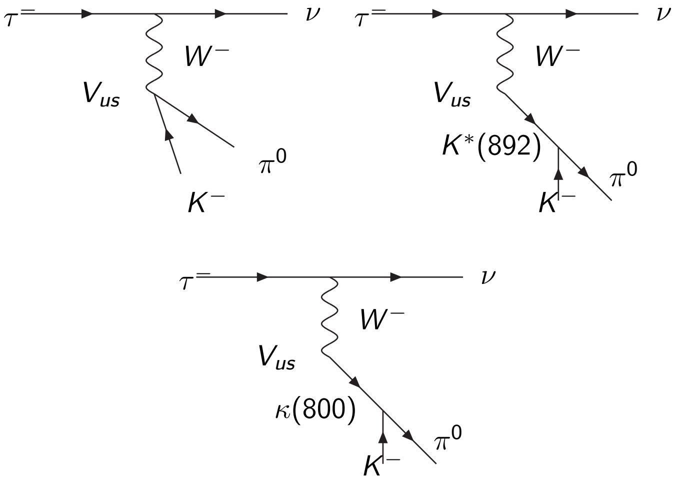
$$\frac{dA_{\rm CPV}}{d\sqrt{s}} \sim \frac{d\overline{A}_{\rm FB}}{d\sqrt{s}} - \frac{dA_{\rm FB}}{d\sqrt{s}} \sim {\rm Im}(F_V F_S^*) \sin \delta_{\rm New} \sim \sin \delta_{\rm st} \sin \delta_{\rm New}$$

- Within the standard model, there is no CP violation because the vector part and scalar part has common weak phase V_{us}.

Calculation of the form factors F_V and F_S.

- Chiral perturbation is not valid when $\sqrt{s} \geq 800$(MeV)
- Inclusion of effects of resonances $K^*(892) : 1^-$, $\kappa(800) : 0^+$ or higher resonances is important. Here we asuume κ is a simple $\bar{s}q$ bound state.
- The chiral Lagranigian including vector and scalars:
  The hadronic Model: Vector and scalar nonets:$(\mathrm{SU}(3)_f)$

Feynman Diagrams



Effective Chiral Lagrangian including scalar and vectors

$$\begin{aligned}
\mathcal{L} &= \frac{f^2}{4}\mathrm{Tr}DUDU^\dagger + B\mathrm{Tr}M(U+U^\dagger) \\
&+ \mathrm{Tr}D_\mu S D^\mu S - M_\sigma^2 \mathrm{Tr}S^2 \\
&- \frac{1}{2}\mathrm{Tr}F_{\mu\nu}F^{\mu\nu} + M_\rho^2\mathrm{Tr}(V_\mu - \frac{\alpha_\mu}{g})^2 \\
&+ \frac{g_1}{4}\mathrm{Tr}(D_\mu U D^\mu U^\dagger)(\xi S \xi^\dagger) \\
&+ g_2 \mathrm{Tr}\left((\xi M \xi + \xi^\dagger M \xi^\dagger)S\right) \\
U &= \xi^2 \quad \xi = \exp(i\frac{\pi}{f}) \\
\pi &= \begin{pmatrix} \frac{\pi^0}{2} + \frac{\eta_8}{2\sqrt{3}} & \frac{\pi^+}{\sqrt{2}} & \frac{K^+}{\sqrt{2}} \\ \frac{\pi^-}{\sqrt{2}} & -\frac{\pi^0}{2} + \frac{\eta_8}{2\sqrt{3}} & \frac{K^0}{\sqrt{2}} \\ \frac{K^-}{\sqrt{2}} & \frac{\overline{K^0}}{\sqrt{2}} & -\frac{\eta_8}{\sqrt{3}} \end{pmatrix} M = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix}
\end{aligned}$$

Description of vector and scalar nonets

$$S = \begin{pmatrix} \frac{\delta^0}{2} + \frac{\sigma}{2} & \frac{\delta^+}{\sqrt{2}} & \frac{\kappa^+}{\sqrt{2}} \\ \frac{\delta^-}{\sqrt{2}} & -\frac{\pi^0}{2} + \frac{\sigma}{2} & \frac{\kappa^0}{\sqrt{2}} \\ \frac{\kappa^-}{\sqrt{2}} & \frac{\overline{\kappa^0}}{\sqrt{2}} & \sigma_{ss} \end{pmatrix} + S_0 \quad V = \begin{pmatrix} \frac{\rho}{2} + \frac{\omega}{2} & \frac{\rho^+}{\sqrt{2}} & \frac{K^{*+}}{\sqrt{2}} \\ \frac{\rho^-}{\sqrt{2}} & -\frac{\rho^0}{2} + \frac{\omega}{2} & \frac{K^{*0}}{\sqrt{2}} \\ \frac{K^{*-}}{\sqrt{2}} & \frac{\overline{K^{*0}}}{\sqrt{2}} & \phi \end{pmatrix}$$

$S_0 = \frac{g_2 M}{M_\sigma^2}$ $M = Diag.(mu, md, ms)$ (quark mass:$m_u = m_d$)

$$\mathrm{Tr} D_\mu S D^\mu S = \mathrm{Tr}(\partial_\mu S + ig[V_\mu, S])(\partial^\mu S + ig[V^\mu, S])$$

generates $\kappa(0^+)$ and $K^*(1^-)$ mixing.

$$K_\mu^{*-} \rightarrow K_\mu^{*-} - ig\frac{(S_{03} - S_{01})\partial_\mu \kappa^-}{M_{K^*} M_\rho}$$

$SU(3)_f$ breaking for vector and scalar mesons.

$$M_{K^*}^2 = M_\rho^2 + g^2(S_{03} - S_{01})^2 \quad M_\omega^2 = M_\rho^2 = M_\phi^2 \rightarrow \text{not} \quad \text{good}$$

$$\frac{M_\sigma}{M_\kappa} = \frac{M_\rho}{M_{K^*}} \quad M_\sigma \sim 690(\mathrm{MeV})$$

Weak current with hadrons

$$
\begin{aligned}
\overline{u_L}\gamma_\mu s_L &= -\frac{1}{\sqrt{2}}F_K\partial_\mu K^- + \frac{M_\rho^2}{\sqrt{2}g}(K_\mu^{*-} - ig\frac{(S_{03}-S_{01})}{M_\rho M_{K^*}}\partial_\mu\kappa^-) \\
&- i\frac{1}{2\sqrt{2}}\left(K^-\partial_\mu\pi^0(\frac{F_\pi}{F_K} - \frac{M_\rho^2}{2g^2F_KF_\pi}) - \partial_\mu K^-\pi^0(\frac{F_K}{F_\pi} - \frac{M_\rho^2}{2g^2F_KF_\pi})\right. \\
&- i\frac{\sqrt{3}}{2\sqrt{2}}\left(K^-\partial_\mu\eta_8(\frac{F_8}{F_K} - \frac{M_\rho^2}{2g^2F_KF_8}) - \partial_\mu K^-\eta_8(\frac{F_8}{F_\pi} - \frac{M_\rho^2}{2g^2F_KF_8})\right) \\
&- i\frac{1}{2}\left(\overline{K^0}\partial_\mu\pi^-(\frac{F_\pi}{F_K} - \frac{M_\rho^2}{2g^2F_KF_\pi}) - \partial_\mu\overline{K^0}\pi^-(\frac{F_K}{F_\pi} - \frac{M_\rho^2}{2g^2F_KF_\pi})\right)
\end{aligned}
$$

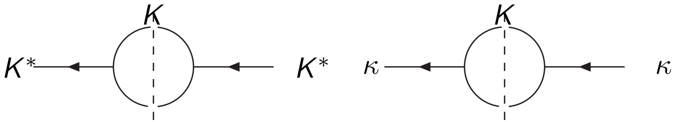
Form factors: Tree level results

$$\langle K^+(p_k)\pi^0(p_\pi)|\bar{s}\gamma^\mu u|0\rangle = \left(g^{\mu\nu} - \frac{Q^\mu Q^\nu}{Q^2}\right) q_\nu F_V(Q^2) + Q^\mu F_S(Q^2)$$

$$\begin{aligned}
F_V^{K^+\pi^0}(Q^2) &= \frac{1}{\sqrt{2}}\left(-\frac{R+R^{-1}}{2} + \frac{M_\rho^2}{2g^2F_KF_\pi}(1-\frac{M_\rho^2}{M_{K^*}^2-Q^2})\right) \\
Q^2F_S^{K^+\pi^0} &= -(m_s-m_u) < K^+\pi^0|\overline{u}s|0 > \\
&= \frac{1}{2\sqrt{2}}\left(-\Delta_{K\pi}(R+R^{-1}) - \frac{Q^2}{M_\kappa^2-Q^2}M_\kappa^2(R-R^{-1})\right. \\
&+ \frac{Q^2}{M_\kappa^2-Q^2}(\frac{m_\pi^2}{2\Delta} - \frac{m_K^2}{1+\Delta})(2\Delta R + (1+\Delta)R^{-1}) \\
&+ \left.\frac{\Delta_{K\pi}}{M_\kappa^2-Q^2}(1-\Delta)(-\frac{m_\pi^2}{2\Delta}R^{-1} + \frac{m_K^2}{1+\Delta}R)\right)
\end{aligned}$$

$$R = \frac{F_K}{F_\pi}, \Delta_{K\pi} = m_K^2 - m_\pi^2, \Delta = \frac{m_u+m_d}{2m_s} \sim \frac{1}{25}$$

The width of K^* and κ in propagators



$P = \pi^0, \eta, \pi^+$ $\qquad$ $P = \pi^0, \eta, \pi^+$

The inverse propagator for K^* is modified as,

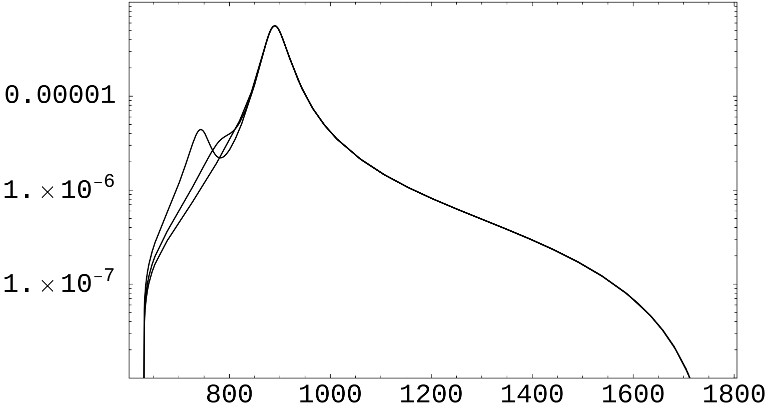
$$
\begin{aligned}
A &= M_{K^*}^2 - Q^2 - iM_{K^*}\Gamma_K^*(Q^2) \\
\Gamma_{K^*}(Q^2) &= 3\frac{1}{48\pi M_{K^*}}\left(\frac{\nu_{K\pi}^3}{Q^4} + \frac{\nu_{K\eta}^3}{Q^4}\left(\frac{F_\pi}{F_8}\right)^2\right) g_{K^*k\pi}^2
\end{aligned}
$$

$$g_{K^*K\pi} = \frac{M_\rho^2}{4gF_K F_\pi} \sim 3.246 \rightarrow \Gamma[M_{K^*}] = 50.8(\mathrm{MeV})$$

$\frac{\mathrm{dBr}}{d\sqrt{s}}$ $(M_\kappa = 750, 800, 850)$

$\mathrm{Br}(\mathrm{K}^+ \to \pi^0 \mathrm{K}^+ \nu)_{\mathrm{th}} = 0.448^{+0.004}_{-0.003}\%$

$1/2\mathrm{Br}[\tau^\pm \to \mathrm{K}_0 \pi^\pm \nu]_{\mathrm{Belle}} = 0.404 \pm 0.002 \pm 0.013\%$



Numerical Results of the vector and scalar form factors

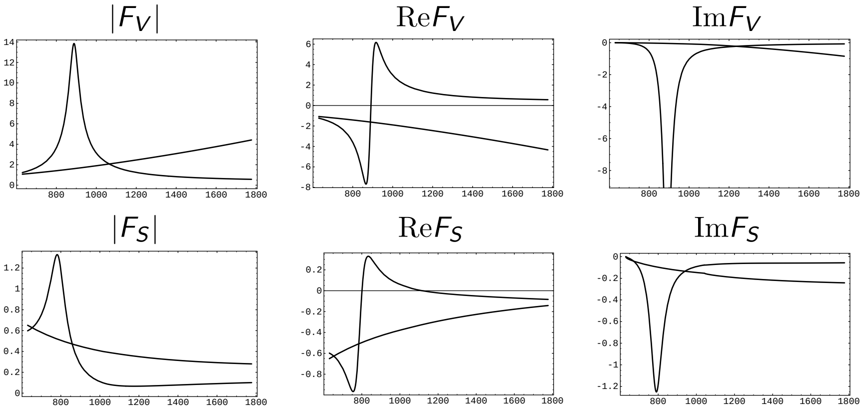
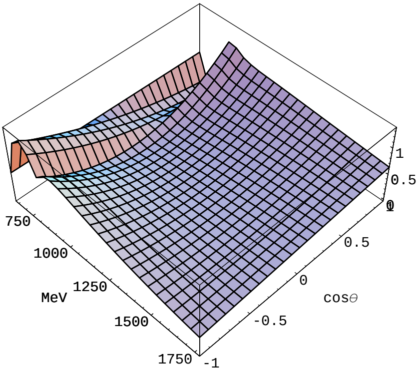


Figure: Form factors (comparison with CHPT)

Double differential rate and Forward Backward Asymmetry



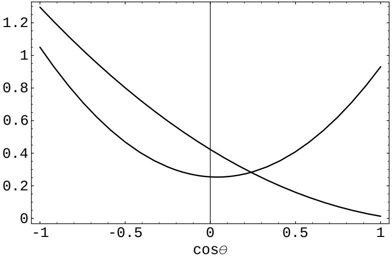


Figure: left. $\frac{d^2\Gamma}{d\sqrt{s}d\cos\theta} / \frac{d\Gamma}{d\sqrt{s}}$ double differential rate

Figure: right. $\cos\theta$ distribution for fixed $\sqrt{s} = 700, 900(\mathrm{MeV})$

Large Forward Backward Asymmetry near the threshold region $\sqrt{s} \sim 700$ (MeV)

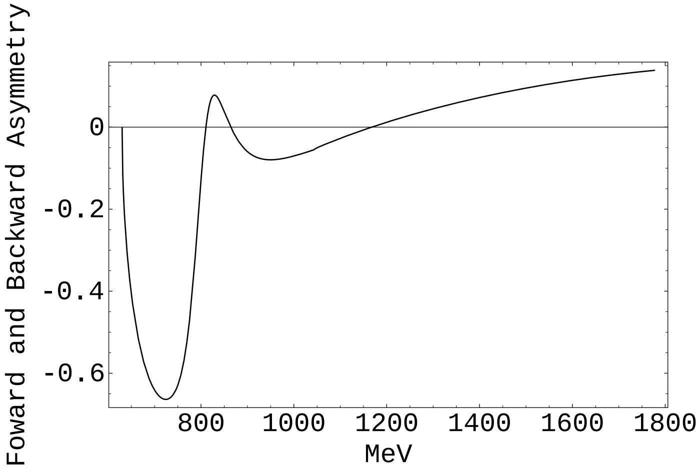


Figure: Foward and Backward Asymmetry

New Physics Effect (Flavor Diagonal New Physics): $\tau_R \longrightarrow \nu_{\tau L}$

$$\mathcal{L} = -\frac{G_F}{\sqrt{2}} V_{us} \bar{\tau}\gamma_\mu(1-\gamma_5)\nu_\tau \bar{u}\gamma^\mu(1-\gamma_5)s + G_{\tau i}\bar{\tau}(1-\gamma_5)\nu_i \bar{u}(1+\gamma_5)s + \mathrm{h.c.}$$

- The 3rd generation flavor diagonal interaction $G_{\tau\tau}$ can interfere with the standard model interaction.
- $G_{\tau\tau}$ is a complex number with a CPV phase differs from the phase of V_{us}.
- How large $G_{\tau\tau}$ contribution ? $a = \frac{\sqrt{2}G_{\tau\tau}}{G_F V_{us}}$

$$\begin{aligned} & \mathcal{M}(\tau^- \to K^-\pi^0\nu) \\ = \quad & -\frac{G_F}{\sqrt{2}} V_{us}^* \left[\left(g^{\mu\nu} - \frac{Q^\mu Q^\nu}{Q^2} \right) q_\nu F(Q^2) \right. \\ & \left. + Q^\mu F_s(Q^2) \left\{ 1 - \frac{Q^2}{m_\tau(m_s - m_u)} a^* \right\} \right] \bar{u_\nu}\gamma_\mu(1-\gamma_5)u_\tau \end{aligned}$$

New Physics interaction and source of CP violation

$$-\mathcal{L} = y_{1ij}\overline{e_{Ri}}\tilde{H_1}^\dagger l_{Lj} + y_{2ij}\overline{e_{Ri}}H_2^\dagger l_{Lj} + y_{2i}^\nu\overline{\nu_{Ri}}\tilde{H_2}^\dagger l_{Li} + \mathrm{h.c.}$$

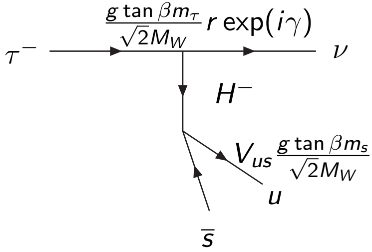
The charged lepton acquires mass from two Higgs y_1, y_2

$$H_1 = e^{i\frac{\theta}{2}}\begin{pmatrix} \frac{v_1+h_1-i\sin\beta A}{\sqrt{2}} \\ -\sin\beta H^- \end{pmatrix} \quad H_2 = e^{i\frac{\theta}{2}}\begin{pmatrix} -\cos\beta H^+ \\ \frac{v_2+h_2-i\cos\beta A}{\sqrt{2}} \end{pmatrix}$$

The relative phase of two Higgs θ (CP violation of Higgs potential.)

$$\begin{aligned} 1-\frac{Q^2}{m_\tau m_s}a &= 1-\frac{Q^2}{m_\tau m_s}\tan^2\beta\frac{m_\tau m_s}{M_H^2}\left(1-\frac{\sqrt{2}M_W}{m_\tau}\frac{y'^*_{2\tau\tau}e^{i\frac{\theta}{2}}}{g\sin\beta}\right) \\ &= 1-\tan^2\beta\frac{Q^2}{M_H^2}\exp(i\gamma)r \end{aligned}$$

Charged Higgs contribution to $G_{\tau\tau} = \frac{G_F}{\sqrt{2}} V_{us} \tan\beta^2 \frac{m_\tau m_s}{M_H^2} r e^{i\gamma}$



Forward and backward CP asymmetry

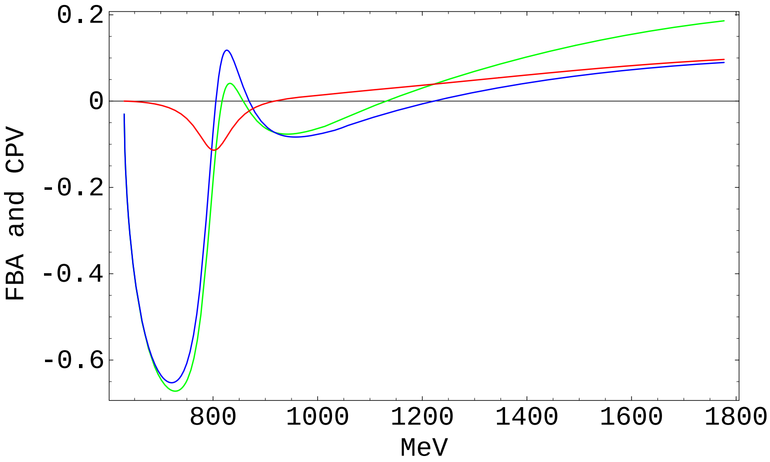
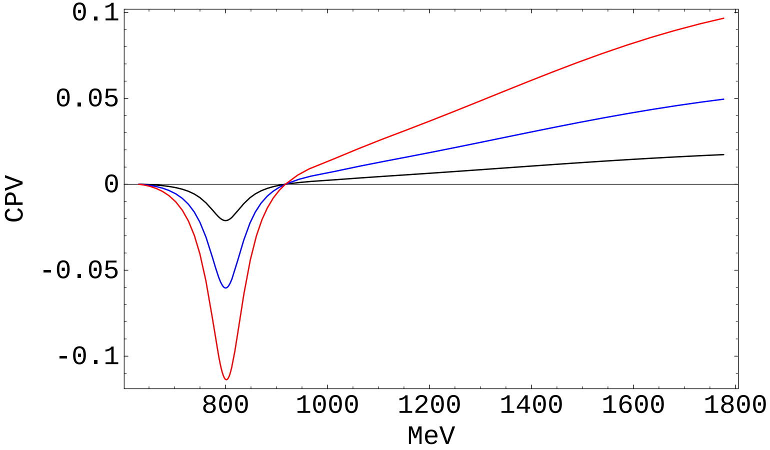
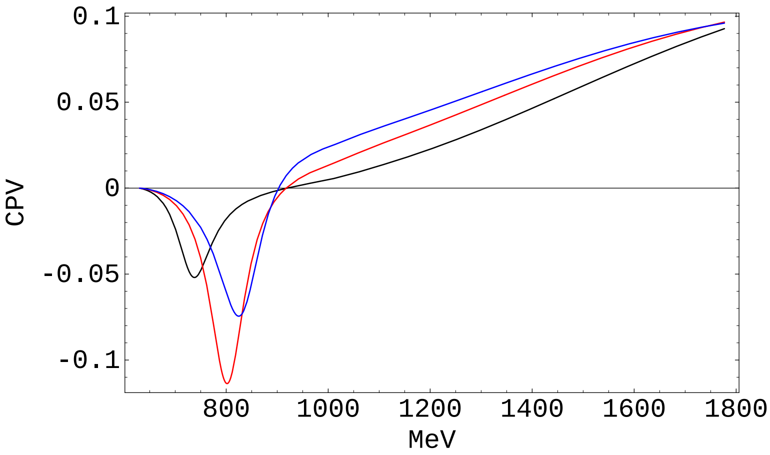


Figure: Forward and Backward CP Asymmetry (Red)
$M_H = 200, \tan\beta = 50, r = 2\ \gamma = \pm\frac{\pi}{2}$

CP phase dependence $\gamma = \frac{\pi}{2}, \frac{\pi}{6}, \frac{\pi}{18}$



Dependence on the mass of κ $M_\kappa = 700, 800, 850 (\mathrm{MeV})$



summary

We study CP violation of $\tau^{\pm} \to K^{\pm}\pi\nu_{\tau}$

- Realistic calculation of the form factors; i.e., We study the vector and scalar form factors including $\kappa(800)$ and K^*. The hadron invariant mass spectrum is obtained.
- We study Forward and backward asymmetry and find the large asymmetry (not CP) $\sim 60\%$ near the threshold region.
- Including the new physics contribution from charged Higgs exchange, we have seen that CPV can be as large as 10%. $(M_H = 200, r = 2, \tan\beta = 50, \gamma = \frac{\pi}{2})$
- CP violating source is identified as from non-minimal $(r \neq 1, \gamma \neq 0)$ type of two Higgs doublet model structure and CPV of the vaccuum expectation value of Higgs.